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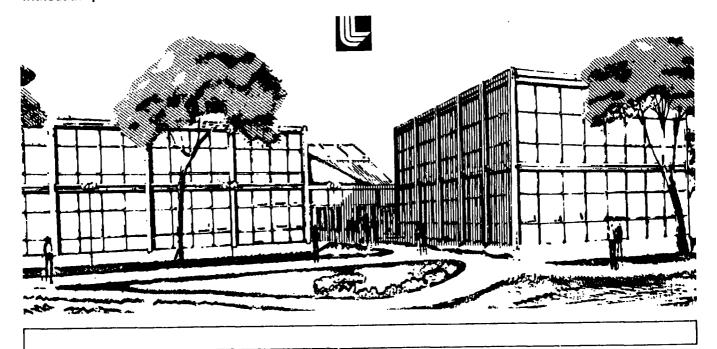


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OPTICAL POLISHING PITCH*

By Norman J. Brown Lawernce Livermore Laboratory

ABSTRACT

This report describes the flow of pitch on polishing laps, discusses the properties of pitch that today appear important, presents some pitch data, and describes pitch testing methods.

INTRODUCTION

Despite three centuries of use, there are significant gaps in our knowledge of optical polishing pitch, the primary medium for polishing in precision optical work today. This report is an effort to close one of those gaps apparently overlooked to date, pitch flow.

In order to put this in perspective, it will review previous work, discuss the properties of pitch, how to measure them, how to control them, and further speculate on their interactions with slurry. This will at times require ranging far afield into discussions of slurry, glasses etc., but since this is a paper primarily intended for optical shop personnel without access to technical libraries, an effort is made to maintain completeness rather than to preserve unities of composition.

The primary media for polishing are almost always viscoelastic, such as pitch, asphalt, and polyurethane, with pitch predominating for precision work. The reason is flow. Most opticians know that at times a wax lap will give a better polish, (although it is prone to orangepeel) and it will hold its figure better, but they seldom use it. Wax is predominantly plastic. A wax ball placed on a table will flatten its base initially but will remain a ball for years, while a pitch ball will deform similarly when it is set down and continue till it becomes a puddle.

This flow affects optical polishing on two levels, macroscopic and microscopic, in control and in bite. On the microscopic level, pitch flow seats the particles of polishing agent. On the macroscopic, pitch flows to mate with the glass, responding to pressure changes. Opticians speak of hard and soft pitches and hard and soft laps. They are not the same. A paper thin lap ungrooved and backed with a rigid mandrel is a hard lap, even with a soft pitch. Such a lap will not effect a curvature change and at the other extreme neither will a lap of jelly. Since control or curvature changes can be obtained between zero values of the extremes there must be an optimum. To find these optima, to be able to scale a process that works well in one case to another, we must quantify our observations. This is the topic of this report.

There must be a more explicit word of caution here than in most reports since many who will read this report are not overwhelmed with literature in their field.

This report is intended to present a picture of current understanding, incomplete and frequently held together by little more than guesswork. An effort has been made to make assumptions explicit, but the possibility of hidden assumptions or phenomena overlooked is always present. However, it has taken us three hundred years to get here, and an industry pressed to the limits cannot wait another three hundred years for the bits and pieces to be welded into a coherent whole. This type of information is needed as early as possible to be integrated into the real, practical research that is a continuing if unrecognized part of the normal optician's practice and trade. The opinions offered are meant only as possible guidelines.

To illustrate my meaning, consider the two optima clearly pointed out herein. I can show that for some processes there is definitely an optimum lap compliance for obtaining a maximum rate of curvature change, (i.e., control), and in several cases I think I know where that optimum is. In no case, however, do I know the shape of the response function, so I don't know how broad the optimum peak is (i.e., how important these results are) nor do I know what effect this has on processes not studied, or even whether the optician wants to work at the optimum. The important point is that we know how to calculate these compliances so the optician has a scaling rule to repeat his results when parameters change.

Again consider Izumitani's clear demonstration (Ref. 1) that there is a distinct optimum pitch viscosity for maximum rates of glass removal. We do not know whether this is also a function of polishing compound properties, or indeed whether the surfaces produced at these rates will be satisfactory for all uses. The important result is that we know there is an optimum and can see if it works to our advantage.

Probably the majority of master opticians have become masters by developing an amazingly intuitive "feel" for these processes. Many already grasp these phenomena in a quantitative way without being able to express them in mathematical terms. Further, many choose to depart from optima either for reasons of a phenomenon not accounted for here, or for reasons of personal preference and technique. The master machinist does not always operate his lathe for the fastest cut, or the deepest, or the smoothest. The results contained here should be checked against practice and if applicable, carefully and cautiously added to working repertoires.

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PREVIOUS WORK

Newton--Probably the original and certainly the first reported use of wood pitch as a polishing medium was that of Sir Isaac Newton (Ref. 2). It was certainly not in general use since he recommended the process to glass lensmakers thirty years later. To polish the first reflecting telescope (using a near relative of speculum metal) he pressed a half millimeter of pitch between two copper spherical mandrels, relying on impressed slurry, (ground and elutriated putty) to prevent adhesion to the upper mandrel. For moisture he used his breath during manual polishing, recommending light pressure and great patience. Seldom have science and immediate practicality been so happily married. The process was used virtually unchanged from that time to this with no reported research until the simultaneous work of Preston and Twyman in early 1920's.

Preston -- In 1922, F. W. Preston published a paper (Ref. 3) on the properties of pitch as a mounting or blocking medium. He showed that the viscosity should lie in a certain range, too high a viscosity preventing strain relaxation of thermal distortions and too low providing inadequate support. He provides an excellent discussion of pellet blocking and of pitch testing. He disparages pointed needle testing as unreliable, preferring truncated cones to these, and not only provides the first reference to a hall test but actually gives the proper form of the equation. He measured his viscosities at 15°C, but it is not clear at what temperature he used them unfortunately. The viscosity range of pitches be recommended lay between 5 x 10^{11} poise and 2 x 10^{12} poise at 15°C. The latter were determined by backtracking from a description of his tests. These appear somewhat high by modern practice, but as mentioned it is not clear at what temperature these were actually worked. Preston refers to wood ash additions to pitch at weight ratios of 1:8, surmising that the particles are separted by thin pitch films, the pitch providing flow relaxation and the wood ash coupling in such a way that its elasticity dominates.

Twyman--During the period Preston was working on pitch at Taylor, Taylor and Hobson, F. Twyman was apparently working on pitch at Hilger Watts (Ref. 4). He likewise recognized that viscosity was the dominant physical property and promulgated the Twyman penetrometer test, the predominant pitch test in most shops today. (This is the penetration in millimeters at 70°F of a 14 millimeter cone truncated to 1/2 millimeter, under a 1 kg load, in five minutes.) He observed and measured the exponential dependence of pitch viscosity with temperature and recommended two pitches, 1-1/2 mm and 3 mm penetrations. He gave several formulae, recommending the addition of one part rosin to eight or ten parts of pitch and from one half to one part of beeswax. He recommended softer pitches with softer materials (e.g., 4 mm pitch with Iceland spar) and observed that wax laps were less prone to sleek but had little tendency to flow. He recommended wax laps composed of two parts rosin 3-1/2 parts of beeswax of some applications. For higher pressure work he recommended stiffening the pitch mixtures above with an additional six parts of willow wood flour (in the same ballpark as Preston's wood ash).

Wada and Hirose (Ref. 5) studied wood pitch and other bituminous materials in torsional creep tests in 1961, and found no significant elastic compliance.

Vinokurov, et al, (Ref. 6) reported that with BF6 glass there was no significant difference in the height of micro irregularities with pitch viscosities ranging from 10^7 to 10^{11} . They reported wax laps produced irregularities 1/2 to 1/3 those found with pitch laps. The following data were reported for 10^9 poise pitch for irregularities.

Particle Size

Glass Type	<u><1 µm</u>	<u>1-3 µm</u>	<u>3-5 μm</u>
K8	105 Å	183 Å	256 Å
BK10	226 Å	310 Å	460 Å
TF1	289 Å	333 Å	497 Å

They also observed that the scratches are most pronounced where polishing compound first contacts the glass.

In a 1971 Hoya report (Ref. 1), Izumitani pointed out the difference between pressing and fitting functions. Using rubber surfaced laps of increasing elasticity, he showed increasing polishing rates (and rougher surfaces with stiffer laps. With pitch and various pitch, pitch-felt, and pitch-rosin mixtures he showed that there is a definite optimum in polishing rate occurring near (2×10^9) poise for SF6 (soft glass) and near (7.5×10^9) poise for BK7. The maxima were rather sharp and a factor of two greater than rates achieved at viscosities an order of magnitude different on either side. Electron micrographs showed that with SF6 glass the less viscous pitches produced finer surfaces.

PITCH PROPERTIES

The dominant property of wood, coal tar, and petroleum based pitches used in optical polishing is their viscosity. This dominates the flow on laps on the macroscopic level and the interaction with the polishing compound particles on the microscopic level.

The property data tabulated in Appendic D shows that viscosities of pitches in common use range over five orders of magnitude, and that these have a strong temperature dependence varying from e $^{-\mu}$ for wood pitches to e $^{-2}$ for petroleum based pitches. Pitch viscosity changes by a factor of two every 1.6 to 2 degrees centigrade for wood pitches and every three degrees centigrade for petroleum pitches (Figure 1).

No implication as to preference is intended in showing only the Gugolz and Cycad pitches in Figure 1. I simply selected two families representing wood and petroleum based pitches. Two batches of Gugolz 82 obtained several years apart are represented as indicated by the triangle orientations. The range of viscosities does not imply that these are all in use at the same temperature. These are intended for mixing as well as for use over an extended temperature range. However several orders of magnitude of different viscosities are certainly used and it is interesting to note that this range is just that which corresponds to the range of geometric compliance effects we shall find later.

Evidence is accumulating that there is an optimum viscosity for each glass, but there is insufficient data at present to determine whether this is primarily a function of the glass alone or whether it is also a function of the polishing compound and lap geometry as well. These topics are currently under investigation. They will be discussed in the section on pitch slurry interactions. It is only important to recognize at this point that material removal rates can be enhanced by a factor of two by selection of the proper viscosity, and that this property also enters in a strong way into the lap compliance function (figure control) in a manner also to be discussed.

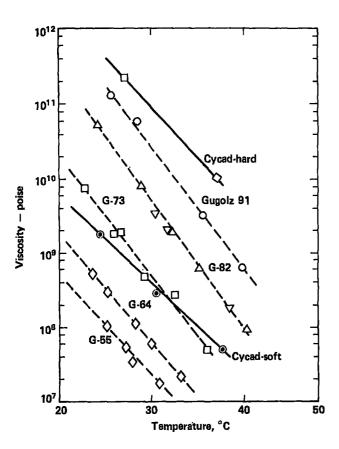


Figure 1: Viscosity as a function of temperature for Gugolz (wood) and Cycad (petroleum) pitches. (Determined by cylinder test.)

Pitch is usually classed as a viscoelastic medium by the rheologists who study such materials (polyurethane is also viscoelastic). This implies that the elastic behavior should be accounted for (Young's modulus) in studies of lap behavior. In 1961, Wada and Hirose looked for elastic compliance in wood pitches and could find no significant effects, concluding that the molecules were too small to form an elastic network. Using more sophisticated equipment and techniques that have become available in the meantime we have found such effects and found them small indeed.

Acoustically determined values of Young's modulus are usually near 2.5 x 10^9 pascal (3.6 x 10^5 ps1).

These and other elastic properties are also tabulated in Appendix ${\bf D}$.

Elasticity turns out to be very difficult to measure because of the degree to which it is overshadowed by viscous flow. Even acoustic measurements turn out to be quite difficult becuase the viscosity tends to dampen and broaden signals and this is complicated by the fact that pitch has an absolutely wild combination of stickiness and brittleness. Experimental students of the subject preparing to use sophisticated equipment should be forwarned that the results tend to perturb rheologists.

There is always a question on the applicability of acoustically measured Young's moduli to slower frequency processes, so we rigged up an oscillating load with a deflection measuring device capable of measuring to microinches. Loading 1.5 cm thick pitch cut into 4 cm square facets with grooves 6 mm deep with an oscillating load varying from near zero to two pounds per square inch at three cycles per minute showed sinusoidal elastic response ranging from one to two microinches, a tenth to a twentieth of a wave in the visible. Since this corresponds to a 50 pound load on a six inch workpiece, we concluded that elasticity effects are probably negligible in normal polishing, when we are considering figuring or the "fitting" function as described by Izumitani (Ref. 1). However, elasticity does have an important effect on surface quality as was clearly demonstrated in the same report where various rubbers, felts, plastics, etc. were studied. This may have an influence on pitch backing selection, and possibly on pitch thickness selection since the range of elasticities we are concerned with is small. Although short time elastic strains are small, they are a function of both pressure and pitch thickness and since they are on the order of the scoring depth of the particle against the glass, they may possibly have a pronounced effect on the bite against the glass (e.g., chatter, slip, etc.

Viscosity on the other hand has a strong effect on figure. Twyman first studied the temperature effects on viscosity finding them to be pronounced to say the least. He correctly assessed the temperature dependence as exponential presenting his data in terms of mobility (reciprocal poise, i.e., $1/\mu$), in terms of a doubling rate:

$$\begin{split} &\text{Mobility} = \mathbf{M} = \frac{1}{\mu} = \mathbf{M}_0 \times \mathbf{2}^{\mathbf{a}(\mathsf{T} - \mathsf{T}_0)} \\ &\ln \frac{1}{\mu} = \ln \frac{1}{\mu_0} + \mathbf{a}(\mathsf{T} - \mathsf{T}_0) \ln 2 \\ &\ln \mu = \ln \mu_0 - (\mathbf{a} \ln 2) \, (\mathsf{T} - \mathsf{T}_0) = \ln \mu_0 - \mathsf{K}(\mathsf{T} - \mathsf{T}_0) \\ &\mu = \mu_0 \, \, \mathrm{e}^{-\mathsf{K}(\mathsf{T} - \mathsf{T}_0)} \end{split}$$

where K = a ln 2.

In this way, Twymans constants for hard and soft pitch transform to (-In 2/1.65 = -0.420) and (-In 2/1.99 = -0.348) respectively, bracketing our own data very well. See Appendix D.

We have investigated the mixing of pitches and find that mobilities rather than viscosities add proportionately, so for the same pitch families we can write for a mixture:

$$\frac{1}{\mu_{\text{mix}}} = \frac{1}{a+b} \left(\frac{a}{\mu_{a}} + \frac{b}{\mu_{b}} \right)$$

where a+b are the weights of pitch type a and type b respectively, and μ are their respective viscosities.

Note that we restrict this relationship to mixtures of the same family. We cannot state that this holds for mixtures of wood and petroleum pitches for example, and it certainly does not hold for additions of waxes and other non viscous materials. Note also that since mobilities add proportionately, the lower viscosity or softer pitch will tend to dominate the mixture.

If we replace the viscosities in the above formula by their reciprocals, the mobilities (i.e., $\rm M_m = 1/\mu_{(a+b)}$ and let $\rm M_a$ represent the lower viscosity material, one part of which is to be added to a material of higher viscosity having $\rm M_b$ represented by b parts to obtain a desired intermediate mobility $\rm M_m$ we can write the above as

let a = 1
$$\frac{(1+b)}{\mu_{\text{mix}}} = \frac{1}{\mu_{\text{a}}} + \frac{b}{\mu_{\text{b}}}$$

$$\frac{1}{\mu_{\text{mix}}} - \frac{1}{\mu_{\text{a}}} = b\left(\frac{1}{\mu_{\text{b}}} - \frac{1}{\mu_{\text{mix}}}\right)$$

$$b = \frac{(M_{\text{b}} - M_{\text{mix}})}{(M_{\text{mix}} - M_{\text{a}})}$$

In our investigations we found that pure pitches or mixtures of pure pitches behave as a nearly pure viscous fluid being strain independent for short times. There is a long time elastic recovery of considerable proportions in terms of optical wavelengths. I have noted that a meniscus several millimeters high, produced against a tape dam, pressed out completely in a short time but when left free standing over a weekend reappeared sufficiently to be easily visible. The inference with regards to laps left unpressed overnight is obvious.

The short time strain independence of both pitches and pitch mixtures of the same family was found to be temperature cycle independent also. Tests run in a Searles type cylinder tester gave the same results whether sheared for a short time or a long time, and completely independent of the temperature cycle sequence. Thus we could test a pitch at three or four temperatures and get the same results for T_2 whether we tested at a lower temperature T_1 first and then at a higher temperature T_3 second, and then at T_2 or if we tested at T_1 first and then went directly to T_2 .

We are publishing no data on wax additions at this time except for this note that small additions of wax (1 to 5%) have a strong effect that is definitely strain dependent and at least exponential with strain. Initially, the pitch wax mixture behaves with a viscosity similar to that of the pitch above. As the pitch flows, its viscosity continuously decreases. The wax molecules are long and can perhaps be viewed as an entangled mass of spaghetti which in shear flow either gradually align with the shearing planes or breakup. Small amounts initially have little effect on macroscopic flow, but as time and shear strain increase, these tend either to lubricate the mixture or to interfere somehow with the self adhesion of the pitch. This raises the rather exciting possibility that a wax proportion could be found that would compensate for geometric variations in compliance of laps where a single pour is used for months at a time, gradually decreasing in thickness.

However, while macroscopic effects are initially negligible, microscopic effects are appreciable. From the very beginning a slight wax addition to a pitch reduces the indentation hardness. The thumbnail test quickly shows the pitch is different. This has unknown implications for pitch slurry interactions where particle indentation or socketing is important.

PITCH PREPARATION

Before leaving the topic of pitch mistures we must discuss the melting and pouring of a pitch. All pitch is a mixture of components, the lighter contributing strongly to lowering the viscosity and being more readily vaporized. In addition to being chemically distinct, these components are chemically and thermally reactive.

During heating the more volatile components rise to the surface and are driven off, increasing the viscosity of the mix. In addition, chemical reactions take place. Particularly important are oxidation reactions, the intake of oxygen being of the same order as that of the fractions evaporated (Ref. 3). These reactions take place fairly rapidly at temperatures above 100°C and are very pronounced at temperatures of 140°C-150°C. These effects presumably occur at a lower rate at lower temperatures so prolonged heating should be minimized. The effects can be reduced by using small deep pots for pitch kept molten during the day (since this minimizes surface area), and by keeping this pot lightly covered (lightly to prevent an explosive rupture) since this will saturate the atmosphere immediately above with volatiles retarding both evaporation and oxidation.

There will however be segregation in such pots and the mixture should be stirred slightly just before pouring. On large laps, I have seen gross segregation where mixing did not immediately preceed pouring. The mixing should be gentle and at a fluidity that permits air, entrapped as bubbles by the mixing, to rise uniformly. I have seen gross density variations on laps due to deep patches of foam.

This segregation in the pots also occurs in large cans where cooling is slow after a pour. The pitch at the top of a five gallon drum tends to be softer than that at the bottom and this is even more the case in a barrel. I find it hard to believe that

there can be much flow in a barrel of hard pitch, preferring to believe this occurs during cooldown, but I have several times heard from master opticians in different shops that the process continues slowly even in cold barrels, so some shops periodically turn their stock containers. However, the possibility of viscosity variations not only with respect to source and batch processing but even with respect to the same barrel and the same cooker is such that periodic testing in the shop by the methods discussed in Appendix A appears warranted.

PITCH FLOW: Macroscopic Behavior

The macroscopic flow characteristic of a viscous lap we will term compliance, defining this as rate of change in thickness of the lap in response to a unit of pressure.

We have mentioned that a paper-thin, ungrooved, rigidly backed pitch lap is a hard lap regardless of the softness of pitch. Similarly the ridiculously exaggerated grooves of the high compliance lap cross section shown in Figure 2 would make it a soft lap regardless of the hardness of the pitch. Even a little pressure will make the pitch flow laterally into the grooves. Even a slight flow here along the large groove face area will produce such a large volume change that to preserve the facet volume the surface must drop significantly in terms of optical tolerances. Since laps at the extremes of hardness and softness (i.c., diamond and jelly) cannot effect curvature changes, this property which we term control must have an optimum if we could find it, and once found we need a descriptive formula to find it again for other cases.

We will find it to be a function of both geometry and viscosity and we need the relationship to compensate for a change in one by a change in the other. Geometry after all will be affected by the size and character of the work, and the pitch properties will be selected for the material to be worked.

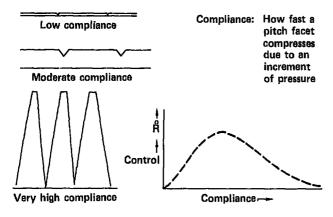


Figure 2: Cross sections (exaggerated) of laps of various compliances. The control-compliance curve simply indicates a guess at its features, zero extremes and at least one ontimum.

While pitch is obviously a very complex medium, it appears that its flow is predominantly Newtonian

(i.e., that of a very viscous fluid). This implies a relatively simple model, but its behavior is far from simple in detail. My rather naive early experiments quickly showed the phenomena were of a complexity that required analytical rather than experimental unraveling. This has been amply verified by the subsequent analysis. As a result, this section will be more mathematical in tone than the other sections of this paper, but the bulk of the mathematics will be reserved to an appendix to preserve readability.

Since most opticians are unfamiliar with hydrodynamics, I should briefly describe a Newtonian fluid. This is one in which a constant shear stress results only in a linear velocity change in the direction normal to the area over which the shearing force acts.

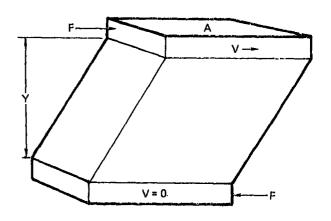


Figure 3: Newtonian flow.

It two plates of area A are coupled to an adhering Newtonian fluid and subjected to a force couple as shown in Figure 3, one plate will move with respect to the other at steady state with a velocity ν in the direction of the force F such that there is no change in the separation y. We can define a shear stress τ in terms of the fluid viscosity μ such that here

$$\tau = \frac{F}{A} = \frac{\mu V}{y}$$

In this simple case, the fluid can be viewed as an assembly of infinitesimal plates parallel to the surface plates sliding over each other with linearly increasing velocity as the plates progress from the lower reference surface to the upper surface.

Immediately, the statement that pitch acts primarily as an incompressible Newtonian fluid dispels one common misconception of pitch flow.

Pitch facets are shaped by separating grooves. A shearing force acting on the surface of the pitch will distort the groove shape but leave the groove volume unchanged, since the groove width will not be affected by pure shear of this type. Since the groove volume remains the same, that of the facets does also, so there can be no thickness change due to this shear.

We are concerned rather with the flow leading to height changes of the facets and consequently to figure

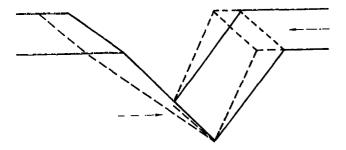


Figure 4: Surface shear induced distortion leaves the groove width unchanged.

changes. These may be mathematically uncoupled, studied separately, and then combined, although the recombination need not be studied to understand the phenomenon. We are interested in how pitch flows vertically.

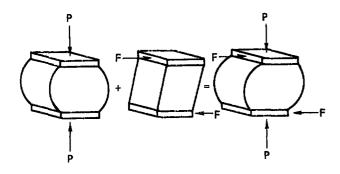


Figure 5: Mathematical coupling of pressure induce shearing and surface shearing.

The pitch flows from the top surface into the grooves rather than being abraided away, although a slight amount of abrasion undoubtedly takes place uniformly over the surface.

The next step in the development of our model is to look at a one dimensional case, a strip W wide and H thick. To study this we will place two strips on top of each other and press them together. This simulates the case on the lap where the pitch adheres to the backing plates but is free to flow laterally on the upper surface which is said to be free of tractions. The model simulates this very well since by symmetry the imaginary platelets on either side of the central horizontal dividing line are moving together producing no horizontal shear with respect to each other across this line. For exact modeling we will assume the pitch is very thin in comparison to its width, much thinner than shown in the diagram, so thin in fact that the pressure across the pitch can be assumed to be a constant across the thickness at any point although it will vary slightly from point to point across the width. From symmetry we need only examine half the width. The pressure distribution is plotted across the median on a vertical scale. Between any two points such as x_1 and x_2 there will be a slight pressure difference AP.

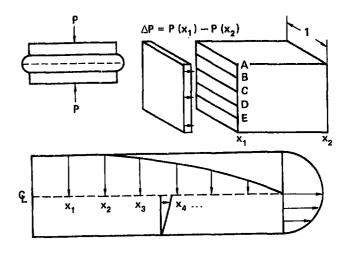


Figure 6: Strip flow model.

For a unit length of this strip, the shearing force at any point of height y, will be

$$F = \Delta P \times 1 \times (H-Y) = \Delta P \times Area$$

For equispaced points A, B, C etc., the shearing force at C will obviously be twice that at B, at D three times that at B, at E four times etc. This will yield a constant change in the slope of the velocity distribution with each increment of height. Note that this is a change in the slope of the velocity distribution, not just a change in the velocity. Such a distribution is parabolic.

Next consider the velocity at point x. The average amount of material flowing horizontally past point x in time interval Δt will be the thickness decrement ΔH occurring in that time increment times the distance from the centerline to point x. At x_2 there will be twice the material flowing past at x_1 , at x_3 three times the material flowing past at x_1 etc. By a similar analysis this leads to a parabolic pressure distribution. The mathematical details are provided in Appendix C. Letting H represent the rate of thickness change per unit time this leads to a pressure distribution

$$P(x) = \frac{3}{2} \frac{\mu \hat{H}[(W/2)^2 - x^2]}{H_3}$$

The optician is only interested in the average pressure \overline{P} which for a strip turns out to be 2/3 the maximum pressure (i.e., at x=0) so we can write,

$$\mathring{H} = 4 \frac{\overline{P}H^3}{10\overline{M}^2}$$

There is a second effectively one dimensional case that can be analyzed, the circular plate. By a similar analysis also contained in the appendix, the pressure distribution becomes

$$P(R) = \frac{3}{4} \frac{\mu \mathring{H}(R_6^2 - R^2)}{H_3}$$

$$\mathring{H} = \frac{32}{3} \frac{PH^3}{\mu D^3}$$

The constants are slightly different from minor differences in integration and averaging. A quick dimensional check shows

$$P(x) = \frac{3}{2} \frac{\mu \mathring{H}}{H^3} \left[\left(\frac{W}{2} \right)^2 - x^2 \right] ; \stackrel{\mathring{H}}{H} \frac{\mu}{P} = C_{\text{strip}} \left(\frac{H}{W} \right)^2$$

$$Circle$$

$$P(r) = \frac{3}{4} \frac{\mu \mathring{H}}{H} \left[\left(\frac{W}{2} \right)^2 - x^2 \right] ; \stackrel{\mathring{H}}{H} \frac{\mu}{P} = C_{\text{circle}} \left(\frac{H}{W} \right)^2$$

$$\mathring{\mathbf{H}} \times \frac{\mathbf{1}}{\mathbf{H}} \times \mu \times \frac{\mathbf{1}}{\mathbf{p}} = \left(\frac{\mathbf{cm}}{\mathbf{sec}}\right) \left(\frac{\mathbf{1}}{\mathbf{cm}}\right) \left(\frac{\mathbf{dynesec}}{\mathbf{cm^2}}\right) \left(\frac{\mathbf{cm^2}}{\mathbf{dyne}}\right)$$

Thus we see that we have the proper form of the equation and the equation for any facet shape can differ only by a constant or by some function of dimensionless ratios.

These results are extremely interesting, showing that these simple cases have a temporal dependence of height varying as the thickness cubed and the facet size squared, strong functions indeed. Further, the viscosity function while shown as linear must be taken in the context of a temperature exponential.

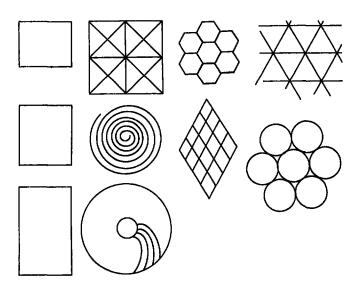


Figure 7: Facet shapes commonly found on pitch laps.

Now these simple cases are a far cry from the number of cases of interest. Facet shapes commonly found on laps hand cut, are usually squares and equilateral triangles. To avoid some of the effects of four-fold symmetry leading to astigmatism, some people further cross cut squares across the diagonals leading to 45-90 triangles, and molded hexagon pressing mats are also frequently used. Diamond shapes are used in special applications, and for cylindrical work and astigmatism removal various degrees of rectangles

are used. The problem to be next attacked is the transfer from the two simple cases above, to the shapes commonly encountered, still keeping in mind that as yet we are working only with very thin laps grooved all the way through to the backing plate.

Those familiar with heat transfer will immediately note a similarity between the heat flow equations for uniform internal heat generation (i.e., resistance heating, nuclear power generation etc.) and the pressure distribution formulae above

$$\frac{\text{Lap Pressure}}{\text{Strip}} \qquad \qquad \frac{\text{Internal Heating}}{\text{Slab}}$$

$$P(x) = \frac{3}{2} \frac{\mu \mathring{H}}{H^3} \left[\left(\frac{W}{2} \right)^2 - x^2 \right] \qquad ; \quad T(x) = \frac{O}{2K} \left[\left(\frac{W}{2} \right)^2 - x^2 \right]$$

$$\text{Circle} \qquad \qquad \text{Cylinder}$$

$$P(r) = \frac{3}{4} \frac{\mu \mathring{H}}{H^3} \left[\left(\frac{W}{2} \right)^2 - r^2 \right] \qquad ; \quad T(r) = \frac{O}{4K} \left[\left(\frac{W}{2} \right)^2 - r^2 \right]$$

By analogy, temperature corresponds to pressure. The heat generated per unit volume corresponds to the increment of incompressible pitch pressed into each volume increment by the squeezing plates, and the reciprocal of the viscosity (i.e., the mobility corresponds to the thermal conductivity. Thus by simply equating

$$\frac{3\mu \mathring{H}}{H^3} \rightarrow \frac{O}{H}$$

we can utilize the heat transfer literature and computer programs that are readily available to obtain the constants C and \overline{P}/P_o to obtain solutions to our class of equations which are of the form

$$\frac{\mu \overset{\circ}{H}}{DH} = C \left(\frac{H}{W}\right)^2$$

These are tabulated in Table 2. Plots of isobars for the shape are given in Appendix C together with an analytical solution for squares, rectangles and equilateral triangles. The latter are especially valuable to computer users who require check cases for mesh sizes etc. Here $Z = (C \overline{P}/P_o)_{shape}$ (C $\overline{P}/P_o)_{strip}$.

Thus far we have been restricting ourselves to very thin pitch cut all the way through to the plate. To look at more reasonable thicknesses again cut all the way through to the plate we need the computer. For those interested in pursuing computer studies on the subject I should point out that going from known displacements backward to pressure distributions is considerably more difficult than proceeding from known pressure distributions to displacements. It is similar to the difference between differentiation and integration. As a result we have so far only computer solutions for the two one dimensional cases treated thus far, the strip and the circle, but as we shall see, these are probably sufficient. We shall

Table 2

С	P/P _Q	$Z = \frac{(C P/P_a) \text{ shape}}{(C P/P_a) \text{ strip}}$	Dimension (W ₀)
25.60	0.441	4.234	Short leg
26.74	0.449	4.502	Leg
12.30	0.465	2.154	Leg
13.39	0.492	2.470	Diagonal
10.67	0.500	2.000	Diameter
9.496	0.476	1.695	Side
7.094	0.483	1.285	Short side
5,838	0.501	1,097	Short side
5.153	0.530	1.024	Short side
4.752	0,563	1.003	Short side
4.0	0.667	1.000	Width
	25.60 26.74 12.30 13.39 10.67 9.496 7.094 5.838 5.153 4.762	25.80 0.441 26.74 0.449 12.30 0.465 13.39 0.492 10.67 0.500 9.496 0.476 7.094 0.483 5.838 0.501 5.153 0.530 4.762 0.563	25.60 0.441 4.234 26.74 0.449 4.502 12.30 0.465 2.164 13.39 0.492 2.470 10.67 0.500 2.000 9.496 0.476 1.695 7.094 0.483 1.286 5.838 0.501 1.097 5.153 0.530 1.024 4.762 0.563 1.003

expect to see considerable difference between the

results calculated by our simple formula when the thickness is appreciable with respect to the width or diameter and steadily decreasing as the thickness decreases. The reason is that in the case of a thin pitch the material which is as it were infused into the spreading layer has a chance to become spread the spreading layer has a chance to become spread throughout the layer (a hidden assumption in our analysis) before it has traveled very far in proportion to the width. In a thick layer the "infused" material does not get so well distributed in producing velocity effects. In Table 3 we tabulate a measure of this which will be $\overline{P}_{C}/\overline{P}_{f}$ (the ratio of the mean pressure from the computer divided by the simple formula value). In addition, we will tabulate $(\overline{P_c}/\overline{P_fc}^{-1})$ in percent. This is the error in percent of the corrected formula as described in the next few paragraphs with respect to the computer determined mean pressure. The independent variable W/H is the ratio of the strip width or diameter to the pitch thickness. Again remember, we are still working with pitch cut all the way through to the plate and actually cut with a verticle cut, not a tapered V. Analysis like everything else has to proceed from a walk to a run. These data are plotted in Figure 8.

Table 3

<u>M/R</u>	<u>2</u>	<u>4</u>	<u>8</u>	<u>16</u>	<u>32</u>	<u>64</u>	128
P _c /P _f Strip	5.292	2.019	1.223	1.040	1.010	1.002	
(Pc/Pcf-1)%	90	68	~1.5	94	19	04	
P _c /P _f Circle							
(P_/P_f-1)%	42	1.6	.50	61	61	. 19	01

The first point to note is that the computer (which can be believed in this case) always provides pressures higher than the simple formula predicts (i.e., lower compliance), especially so in the case of small width to thickness ratios. The pitch flow is taking place much closer to the surface particularly in the center region of the circle, instead of distributed across the full thickness everywhere. As a result in a 2:1 diameter to thickness ratio the simple formula predicts a pressure almost an order of magnitude low. In the normal range of facet sizes, 8:1 to 2:1 the error ranges from 50 percent to 1000%.

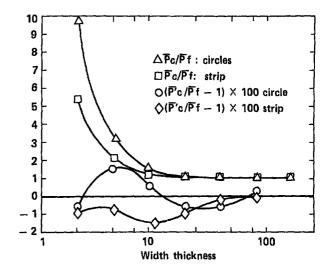


Figure 8: Relationship of detailed computer run values to simple formula values, and the error resulting from the modified formula. Note that the error is magnified by 100 times.

In trying to bring order out of confusion I noted that plotting $1-\overline{P}_C/\overline{P}_f$ against W/H on log-log paper yielded an almost straight line with a slope near -2. The function of dimensionless ratios was almost certainly a series and the coefficients should be near whole numbers. Without going into details the first three series terms heuristically pulled out turned out to be

$$\left(1 + \frac{20}{(W/H)^2} - \frac{2}{(W/H)^{1+6}}\right)$$

for the strip and

$$\left(1 + \frac{40}{(W/H)^2} - \frac{4}{(W/H)^{1.6}}\right)$$

for the circle. The exponent 1.6 is probably an approximation term for the remainder of the series, but the computer results were obtained with a mesh size consistent only with four digit accuracy so further reduction was not warranted. The error obtained by multiplying the $(\text{W/H})^2$ factor in the simplest formula by this additional factor which we shall call F_1 (W/H) is shown in Table 3 and Figure 8 showing it has some validity. The exact factor of two difference in the two series coefficients is just too much to be fortuitous. At first I believed it was the Z factor listed in Table 2, but on examining the pressure variations in the vicinity of the edge (see Appendix C figures) I have come to the conclusion that this is due to a factor we shall term Z defined as

$Z = \frac{\text{(edge area/facet surface area)}_{\text{shape}}}{\text{(edge area/facet surface area)}_{\text{strip}}}$

Thus the coefficients in $F_1(N/H)$ above become 20Z and 2Z, where Z = 1 for the strip and Z = 2 for the circle.

From our analysis thus far it should be clear that each of these ratios can be translated into an orifice area (edge length x groove depth) to flow volume (A x facet surface area) ratio. Comparing the circle of diameter W and strip of width W we obtain an exact factor of two thusly

$$Z_{circle} = \frac{(\pi Wh)/(\pi W^2/4)}{(2Lh)/(WL)} = \frac{4h/W}{2h/W} = 2$$

At this point we certainly have an equation that predicts compliances for full vertical cut strip and circle facets to a very close approximation, and we can probably extend this to any of the other shapes as well simply by using the appropriate Z factor defined above in $F_1(W/H)$.

Now we must look at the case of grooves only partially cut through. Tables 4 and 5 tabulate for strips and circles respectively GS and GC which are the dimensionless compliances we shall define as

In these tables each number is followed by the exponent of ten by which the number must be multiplied (i.e., $1.855-1 \approx 1.855 \times 10^{-1}$.

Plotting the data of Table 4 on log-log scales in Figures 9 and 10 reveals that the groove depth to thickness ratio effect is quadratic as shown by the way the points align with the heavy dashed line drawn with a slope of two.

However, when we simply modify the formula we already have by a factor $(D/H)^2$ we find the errors plotted in Figures 11A and 11B. The function required will not be a simple one as shown by the way the errors decrease in both figures as (W/H) varies from two to eight and then begin to increase again. Furthermore, the fact that D and H are identical for the full depth vertical cut case leads us to question which if any of the W/H factors there should really be written as W/D.

Since the function F_1 appeared to be a measure of flow distribution through the thickness, I decided to leave the ratios as W/H in the series, and faced with a limited time budget I finally brute forced the vertical cut groove case into

$$G = C \left[\frac{D^2}{W^2} \right] / \left[F_1 \left(\frac{W}{H} \right) \times F_2 \left(\frac{D}{H} \right) \right]$$

where C is the value from Table 2 and

$$F_{1}\left(\frac{W}{H}\right) = \left[1 + 20Z \div \left(\frac{W}{H}\right)^{2} - 2Z \div \left(\frac{W}{H}\right)^{1.6}\right]$$

$$F_{2}\left(\frac{D}{H}\right) = \left[\left(1 - K\right)\left(\frac{D}{H}\right)^{2} + K\left(\frac{D}{H}\right)^{L}\right]$$

$$K = 1 - \left(0.6Z + 0.8\right) \div \left(\frac{W}{H}\right)^{3} - \left(0.3 - 0.07Z\right)\left(\frac{W}{H}\right)^{B}$$

B = (0.35+0.09Z)
L =
$$-\left[\ln\left(\frac{W}{H}\right)\right]$$
 + (12-3Z) + 1 + (24-10Z)

Table 4

	<u>D/H →</u>	1.00	0.80	0.60	0.40	0.20	0.10	0.05
	W/H = 2							
	ES GC	1.855-1 96 2.766-1 .42	28 2.158-1	85 1.511-1	5.079-2 .61 8.554-2 4.0	1.9 2.677-2	1.2 7.082-3	-2.5 1.749-3
	W/H = 4			•				
	ES GC	1.239-1 .69 2.141-1 -1.6	1.7 1.635-1	-1.0 1.060-1	-3.4	-3.6 1.439-2	-1.8	-3.1 8.361-4
	W/H = 8		•					
	ES	5.112-2 1.5 1.119-1 49	4.2 8.826-2	2.0 5.718-2	79 2.778-2	.28 7.163-3	3.9 1.700-3	2.4 3.769-4
W/H = 16								
	ES GC EC	1.502-2 1.4 3.781-2 .62 Se	2.6 3.239-2 2.9	.16 2.325-2 1.3	-3.7 1.237-2	-3.7	-1.4	-6.5
	GC EC	1.024-2 .61	9.390-3 1.6			1.394-3 .06		

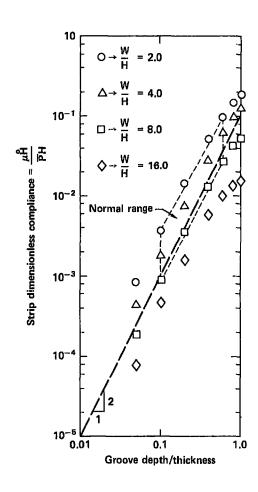


Figure 9: Dimensionless compliance as a function of groove depth to thickness ratio for vertical grooves as determined by computer for infinite strips.

Table 5

(Dimensionless compliance for 45° grooves. This data has not yet been reduced to formulae.)

D/H	1.00	0.80	0.60	0.40	0.20	0.10	0.05
W/H = 2							
GS GS	::	5.734-1 2.571-0	2.739-1 8.525-1	1.136-1 2.970-1		6.427-3 1.474-2	1.490-3 3.349-3
W/H = 4							
GS GC	2.403-1 7.513-1	1.725-1 4.773-1	1.074-1 2.735-1	5.102-2 1.228-1	1.299-2 2.993-2		7.042-4 1.564-3
W/H ≈ 8							
GS GC	6.916-2 1.905-1	5.650-2 1.483-1		2.151-2 5.141-2		1.432-3 3.192-3	3.134-4 6.901-4
W/H = 16	5						
GS GC		1.591-2 4.264-2				6.219-4 1.379-3	1.234-4 2,700-4
W/H = 32	2						
GC		1.114-2	9.515~3	6.653-3	2.351-3	5.291-4	8.865-5
W/H = 64							
GC	2.930-3	2.807-3	2.560~3	2.015-3	8.190-4	1.708-4	2.521-5

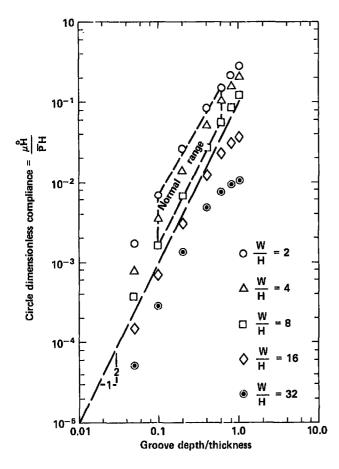


Figure 10: Dimensionless compliance as a function of groove depth to thickness ratios for vertical grooves as determined by computer for circles.

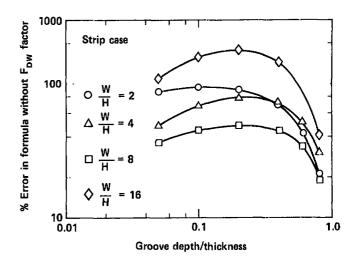


Figure 11A: Error for strip case in modifying formula by multiplying by (D/H)² only.

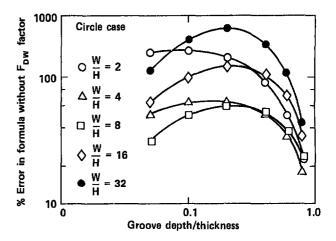


Figure 11B: Error for circular case in modifying formula by multiplying by (D/H)² only.

This fits the data reasonably well as seen by the errors tabulated as ES and EC for the strip and circle in Table 4 and in Figure 12 for W/H values through 16 and can be extended to W/H = 32 by replacing the L above by 0.147. However, while I believe F_1 is a partial representation of reality, I do not believe the combination $F_1 x F_2$ has any deep physical significance. I am publishing it only for those who may need a reasonably accurate representation over the entire range.

However, when we try a less ambitious fitting scheme covering the normal range of values of D/H and W/H in use, we suddenly hit pay dirt. Here a surprisingly simple formula appears to cover both vertical and 45 degree cut grooves for both strips and circles

$$G = 0.7Z (D/H)/(sin^2\theta(H/H))$$

for 0.1<D/H<0.6; 2<W/H<8; 45°<0<90°

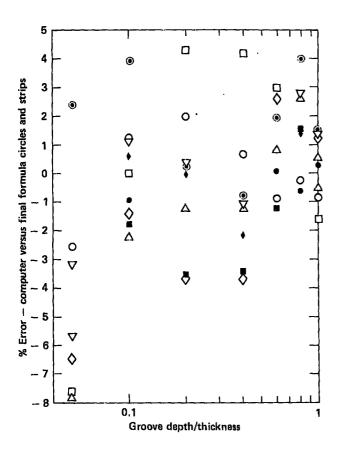


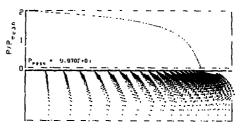
Figure 12: Error for the final formulae as compared to computer results for the vertically cut groove cases. Two data points are omitted from the 0.05 (D/H) values at -10.0 and -11.6%.

yielding a fit usually better than 25% for useful values. The constant was weighted to yield the best values for the circular 45° cut case as the most practical case. 8 is the angle of cut with respect to the surface (i.e., a vertical cut is 90°). The factor Z is that from the orifice area/flow volume interpretation almost certainly.

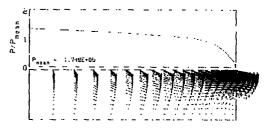
There are two features even beyond the simplicity of this equation that I find rather startling. As we convert from dimensionless compliance to compliance we obtain

$$\frac{\mathring{H}}{D} = \left(\frac{0.7Z}{\mu}\right) \left(\frac{D^2}{W \sin^2 \theta}\right)$$

Note that the thickness H has completely disappeared and W appears only as a linear dimension. The orifice area is proportional to D/sino dominating the equation. The interpretation is that the facet in this range acts as if it is full cut sitting on a layer of pitch and while some flow takes place below the layer defined by the groove depth, the variations here have little effect on the phenomena.



9:1 RADIUS: CHICKNESS, GROOVE CEPIP 0.61



4:1 RADIUG: (HICKRESS RATIO, EO% SROOVE DEPTH

Figure 13: Flow velocity pattern's and pressure distributions for vertical and 45° cut grooves on circular facets. The mean pressure in the 45° cut case is based on a facet area entending to the groove center.

These effects are clearly seen in Figure 13. Note the sharp pressure drop near the edge and the different character of the flow demonstrating the dominance of the orifice flow impedance. Note that the pressure is almost exactly a factor of two less in the 45° cut case indicating a factor of two greater compliance (1/sin² 45° = 2).

Note also that the range of this simplified equation is almost exactly coincident with the range of parameters master opticians choose to work in. They have selected the entire range where absolute thickness effects can be neglected and where the phenomena are sufficiently simple that they can be intuitively grasped and they seldom exceed those limits. I have frequently wondered at what seemed an incredible waste of pitch in the layer below the groove depth. It has consistently been my observation that mathematical analysis may quantify the opticians intuition and clearly point out the factors of importance such as the dominance of the groove depth and shape in this case; however, if it fails to clearly and rationally explain the common practice of master opticians, I am immediately suspicious. There are 300 years of trial and error evolution involved in this practice.

This mathematical detail has been provided because of the insight given into the processes by the form of the equations. Secondly, the equations can be readily programmed into the hand and desk calculators that are rapidly becoming important shop tools. However, in striving for precision I want to be careful not to overstate the case. These data and formulae should be used to seek out process

optima. I do not want to imply that these optima occur as sharply peaked functions such that the optician must know his lap compliance to a few percent to do good work. Such a belief could lead him to an uneconomically long search for a vanishingly small degree of perfection. I suspect that the peak is rather broad, that the acceptable range for a given process is appreciable; otherwise we would not have been able to function competatively for so many years. Nor do I wish to imply that this is the full and exclusive story on pitch flow. Utimately, this data will be used to examine lap curvature changes. Individual facets are not going to be uniformly pressed down in step increments of flatness. Some facets will be pressed down at the edges, some at the centers and others skewed. There will be effects due to pitch surface property changes, usually hardening, due to leaching, particle absorption etc. These surface effects should be small since vertical flow results from movements below the surface but they serve to point up the fact that these functions are in the limit only approximations of reality.

At the same time, I do not want to understate the economic importance of these studies. They show almost conclusively that there are between three and four orders of magnitude of lap compliance in common use today. Now I am not speaking of the three to four orders of magnitude spread of the data. That range was selected to demonstrate the phenomena. I have seldom seen width to thickness ratios (W/H) of 16 hut I have seen plenty at eight and slightly above. I have seldom seen facets cut all the way through or even to 80% depth but I see cuts ranging from 10 to 60% depth in common use and toward the end of life I have seen not a few grooves closed to near 5%. From this I conclude that the dimensionless ratio range is such that dimensionless compliances range slightly over two orders of magnitude marked on the figures as the normal range. Note that this is dimensionless compliance,

$$G = \frac{\mu \overset{\circ}{H}}{PH}$$
Compliance = $\frac{\overset{\circ}{H}}{P}$ = $G \frac{H}{\mu}$

Lab process behavior is predicted on compliance which means that this G must be multiplied by lap thickness which ranges from several millimeters to several centimeters, and divided by viscosity of which easily more than an order of magnitude is in common use. This is the basis for my statement of three to four orders of magnitude.

Now I do not know the shape of any single process response curve at this time beyond the fact that for at least some particular processes at least one and probably only one optimum exists. However, I have frequently heard very emphatic declarations by opticians that such and such a lap is definitely not working optimally and have experienced this myself. It appears quite possible to be well off optimum and I have soldom seen any optimal peak extend anywhere near three to four orders of magnitude.

There are a number of additional points of confusion in my mind with respect to this optimum and these must be clearly stated, a point that seems almost a

contradiction in terms. First, the concept of an optimum is meaningless without specifying the function to be optimized and the variables with respect to which optimization is being pursued. I believe I have been clear in stating that the function described has been rate of change of curvature which I regard as the dominant function described by the term "control". On the other hand for some processes, "control" might mean just the opposite.

Furthermore while we tend to categorize phenomena into compartments, topics of study etc., nature and processes are continuous. This topic cannot be pursued in complete independence of wear function theory. I have recently published a brief article calling attention to flexible laps for aspheric generation and polishing, laps which have a maximum of compliance. How can I speak of such compliant laps being valuable at one time and some much less compliant laps being too soft at another? A brief answer is that wear patterns can at times be primarily dependent on pressure variations across a single pad and at another primarily dependent on dwell time in a certain zone at relatively uniform pressure.

Thus lap compliance is a function of the process, of more importance in some than in others. It will take prolonged intensive study to properly sort these out, and this study hinges critically on the pitch slurry interactions to be described in the next section. I can and have written computer programs that examine wear in terms of both elasticity and viscous flow compliance but they aren't worth the paper they are printed on because they do not properly model the pitch-slurry-glass interface to be treated. These considerations are important to the optician in that they tell him confusion exists. He should not screw up working processes trying large step improvements based on what is at this time only a partial picture.

I cannot at this time describe the best use of this data, (i.e., tell an optician the best way to make a lap for a particular operation). However even beyond providing an understanding of the process, I would recommend the following. Have someone program one of the magnetic cards for one of the small programmable calculators. Keep track of pitch viscosity with the ball test to be described and a notebook of lap parameters used for different jobs with a notation on the lap performance, good, bad etc. Periodically reducing this data, the optician should find he has a key to identify the range of parameters that best suit his particular practice. Once he knows this he should be in a better position to tailor his laps to his own need.

Appendix B contains some further speculation on this subject.

PITCH-POLISHING COMPOUND INTERACTIONS

Very little definitive work has been done in this area, although much very competent effort has been expended. The reason is that the process is so complex that frequently each new bit of information requires a complete rework of previous data since new factors formerly unanticipated are brought in by new observation or speculation.

Prior to beginning a new round of speculation, I will provide a few calculations of the "back of the

envelope" variety in an attempt to limit its range. These are designed to answer the questions of elastic effects and transient penetration of particles into a pitch lap prior to further discussions.

First let us examine the question of elasticity. Assuming a Young's modulus of 3×10^9 Newton's/m² or 3×10^{10} dynes/cm², consider the deflection of a lap. We will consider two cases briefly, one from a single planetary lap and one a typical spindle operation.

For the single planetary case assume a 2 cm thick piece of 2.5 density glass on a 2 cm thick pitch lap. The pressure P will be the product of density, thickness and gravitational constant. This will equal the product of Young's modulus and strain, the latter being the elastic deflection Δ divided by the pitch thickness

$$P = \rho t_{g1}g = \frac{E\Delta}{t_{pitch}}$$
; $\Delta = \frac{2.5x2x2x981}{3x10^{10}} \approx \frac{1}{3} \times 10^{-6} cm$

We can get a feel for this by dividing by the wavelength of visible light,5 x $10^{-5} \, \mathrm{cm}$

$$\frac{\Delta}{\lambda} \approx \frac{10^{-6}}{3x5x10^{-5}} \approx \frac{1}{150}$$

of curvature encountered.

For the single spindle case assume 1 cm of pitch (ρ =1.0) plus 1 cm of aluminum (ρ =2.7) plus 1 kg of arm load. Assume the arm load is distributed over a four inch diameter pad (10 cm).

$$P = \left(3.7 + \frac{10^3}{25\pi}\right) \times 981 = 16.4 \times 981 = 3 \times 10 \quad \frac{\Delta}{1 \text{ cm thick}}$$

$$\Delta \approx \frac{15 \times 10^3}{3 \times 10^{10}} = 5 \times 10^{-7} ; \frac{\Delta}{\lambda} \approx 1/100$$

Thus the elastic deflection is on the order of one hundredth wave, negligible in terms of the changes

Note that in saying elasticity effects on the order of a wave are negligible we are only speaking of figuring. The glass appears to be supported off the pitch by a monolayer of particles varying in shape and statistical orientation and easily able to more or less uniformly support the glass in spite of separation variations of tenths of waves. This is easily possible since mean particle size ranges from half a micron to a few microns, a visible wave to a few waves. If glass were resting right on the pitch or on a layer of particles exactly the same size in all orientations, with this kind of elasticity effect the glass would be contacting only on those areas defined by isobars surrounding peaks on the glass at a few hundredths of a wave of thickness away from the peaks and soon these areas would be sharply defined by fringes breaking away from hundredth wave smoothness. What we actually see is continuous polishing over undulations a moderate fraction of a wave in amplitude with the peaks gradually decreasing due to a combination of effects, the primary one being an increased probability of finding a particle in a position to cut where the gap between pitch and glass is smallest.

At the same time we cannot so easily eliminate elasticity effects from polishing rate or cosmetic quality. While the strains are small on the order of separation of glass and pitch, they are appreciable

in terms of the depth to which a particle is scored by the abrasive. In Hoya Report HGW-0-3E, Izumitani has showed that there is a 50% rise in polishing rate of glasses when rubber and felt elastic shear moduli increase by an order of magnitude and that there is a pronounced difference in the surface roughness in Nomarski micrographs, the harder surfaces producing rougher surfaces. It is easily possible that elasticity effects can influence the bite of the particles into the glass through such mechanisms as influencing chatter, minute shocks etc., as well as force particles through the hydrolized layer into virgin glass.

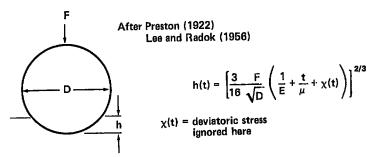
The acoustic velocity in solids is given by the square root of the sum of Young's modulus and 4/3 the shear modulus both divided by the density, yielding a velocity of approximately 2 x 10⁵ cm/second. Acoustic resonances on the order of 10⁵ cycles per second could be set up in pitch on the order of a centimeter thick. Since typical lap velocities are on the order of one to ten centimeters per second, chattering could take place on the order of every particle length of scraping or less. Chicken track chatter marks of about this frequency are frequently seen in glass, the periodicity being set by the rupture strength of the glass and the particle size, and ten kilocycles is about the frequency of squeal heard around a lap that is not always dry. There may be an implication here between lap speed, thickness, and particle size and smoothness but I can't see it clearly. I simply note it so someone else may have a chance to make something out of it.

Returning to the question of figuring and looking at viscoelastic effects, we note that most viscoelastic solutions have a factor of the form (1/E + t/ μ). Since viscosities used are in the range of 10^9 to 10^{10} we see that this typically reduces to

$$(.3 \times 10^{-10} + ... \times 10^{-9}t)$$

showing that the elastic portion of the solution is usually negligible after a few tenths of seconds.

Looking now at the penetration of slurry particles into pitch we find Lee and Radok's 1956 solution (Ref. 7).



This solution is similar to an elastic solution by Timoshenko and Goodier (Ref. 8)

r (t) =
$$\frac{3 \text{ RF(t)}}{16 \text{G}}$$
 where G = $\frac{\text{E}}{2(1+\text{v})}$ = shear modulus.

Making the approximation that R>>h so 2(R-h) $^{\approx}$ D, h = R-d, and d^2 = R^2-r^2 we can manipulate to obtain

$$h(t) = \left[\frac{3F}{16\sqrt{D}} \left(\frac{1}{E} + \frac{t}{\mu}\right)\right]^{2/3}$$

a result very similar to Preston's empiracally derived expression of 1922 (Ref. 3) if the elastic term is dropped.

$$\mu = const \frac{-Ft}{h\sqrt{hD}}$$

Cerium oxide polishing compounds range from 1/2 to 3 microns with a density of 7.132 theoretical. Assume the ultrafine half micron size and a 10% by weight fraction (a distinct optimum as shown by Sergeev Ref. 6, pg. 82). Assuming v is the volume fraction we can proceed to calculate the rate of pitch penetration by a free particle passing between pitch and glass as follows:

wt fraction =
$$\frac{7.132v}{7.132v + (1-v)} = .1$$
; $v = 0.01534$

Assuming uniform size spherical particles the average volume per particle is 6.545 x 10^{-14} cm³ at half micron size. Assuming the space between glass and pitch is the particle size (a result we will find), the volume becomes half a micron per square centimeter of interface yielding with the above 7.67 x 10^{-7} cubic centimeters of ceria and 1.172×10^{7} particles per square centimeter. Assuming a planetary lap with pitch viscosity 2 x 10^{10} poise and a glass 2 cm thick of density 2.5, we find 4.18 x 10^{-4} dynes per particle load and substituting we obtain

$$h(t) = 4.97 \times 10^{-2} \left[\frac{1}{3} \times 10^{-10} + \frac{1}{2} \times 10^{-10} t \right]$$

yielding 1.9 x 10^{-12} cm in 0.1 sec and 4.12 x 10^{-12} cm in one second.

This result is important because it completely dispels the possibility that edge roll-off is due to particles rapidly sinking on the order of a micron (the magnitude of edge round off) into pitch in the first centimeter or so that they travel between pitch and slurry, a point we shall return to in further discussions of edge round off.

An interesting side result is the following: We have just shown that glass and pitch are probably separated by the diameter of the slurry particle D, so the interface volume is probably D cm 3 per square centimeter of interface area. Thus for a given concentration of slurry the number of particles is inversely proportional to D 2 since

$$N \propto D/\left(\frac{4}{3}\pi R^3\right) \propto 1/D^2$$

Thus the force per unit particle is proportional to D^2 . Replacing F in the Lee and Radok formula we obtain

$$h(t)_{pitch} = const \left(\frac{D^2 t}{\mu \sqrt{D}} \right)^{2/3} = const \left(\frac{t}{\mu} \right)^{2/3} D$$

Thus for a given time and pitch viscosity h(t)/D is a constant. This says that the fraction of its diameter that a particle sinks into a pitch in a given

time is independent of size. Furthermore the fraction of its diameter that it penetrates into the glass due to elastic effects will also remain the same as can be seen simply by replacing the t/μ term in the Lee and Radok formula by 1/E.

$$h(t)_{glass} = const \times \left(\frac{1}{E} \frac{D}{\sqrt{D}}^{2}\right)^{2/3} = const \left(\frac{1}{E}\right)^{2/3} D$$

Thus for a given concentration, time, and pitch we have the following:

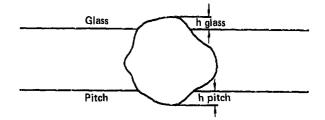


Figure 14: Particle penetration in pitch per unit time and into glass should be a fraction of its diameter independent of its size for a specified slurry concentration.

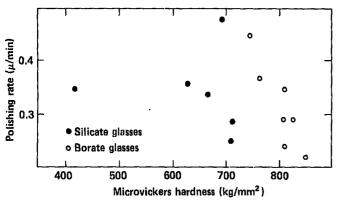
The glass and pitch penetration or socketing proportions should be the same independent of particle size. Thus the socketing function relationship to the moment arm should remain the same. The width of the scraping scratch and the depth should be proportional to the size of the particle. Note that this may be exactly what Vinokurov found. Columns 2 and 3 (of Table 1) of his data agree with this if we assume that the larger particles are contributing most of his observations. Further since the frontal area fraction of the particle opposing the glass (i.e., immersed in the glass) varies as the square of the particle size, but the number of particles per unit area of interface varies inversely by the same square, it is quite possible that volume removal rates are somewhat independent of particle size. This is somewhat unclear in my mind since other effects could influence the last statement (i.e., absolute penetration rate into the pitch away from the glass varies as the diameter etc.), but the effect should be pointed out since it may be an important clue for further research, and since it certainly has significant implications for those seeking ultra smooth surfaces.

With these questions out of the way, let us examine the observations. Preston observed that polishing rate was proportional to pressure and velocity. Vinokurov's observations lead us to believe that particle size may be important, particularly maximum particle size.

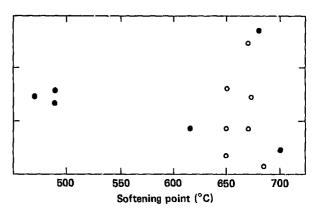
Izumitani, with a controlled glass supply, found no correlation between polishing rate and microvickers hardness of the glass, indicating that polishing is not a simple glass cutting function. Similarly he found no correlation between glass softening temperature and polishing rate tending to eliminate glass flow. However, when he plotted polishing rate against the weight loss of glasses immersed in .01 normal nitric acid and against microvickers hardness of glasses

leached for 60 minutes in 0.1 normal nitric acid he found two rather striking linear relationships for silicate glasses and for borate glasses. He also showed that this polishing rate continues after the surface becomes smooth, and that polishing rate is much slower in oil slurries than in water slurries. This beautiful sequence of experiments demonstrates almost conclusively that polishing takes place by abraiding a layer softened by hydrolysis, and furthermore that the rate of polishing may be dependent on the rate of formation of the hydrolized layer.

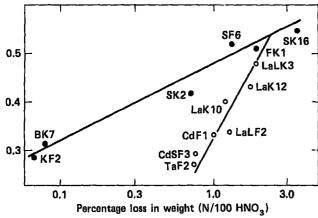
Before proceeding with pitch slurry interactions I would like to compare two sets of observations on cerium oxide relating the rate of polishing to the firing temperature of cerium oxide. In 1945, Davis and Wayman reported that cerium oxide fired below 1050°C was too soft to polish glass and that fired above 1170°C was too hard, continuing to polish glass but leaving a roughly scored and abraided surface. Repeating this type of test a few years ago, Izumitani obtained quite different results shown in Figure 20. Except for the very definite temperature shift, the



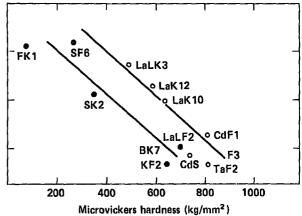
Relationship between microvickers hardness and polishing rate in water polishing.



Relationship between softening point and polishing rate in water polishing.



Relationship between glass polishing rate and weight loss after being soaked in N/100 nitric acid solution.



Relationship between polishing rate and microvickers hardness after leaching for 60 min. in N/10 hydrochloric acid solution.

Figures 15, 16, 17, & 18: Results of Izumitani experiments from Ref. 1.

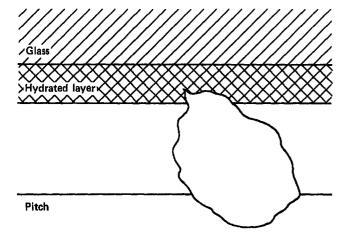


Figure 19: Polishing takes place in a hydrated glass layer.

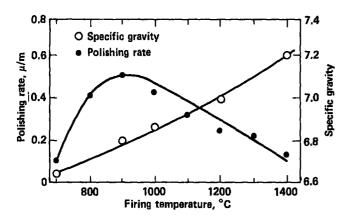


Figure 20: Polishing rate as a function of cerium oxide firing temperature - Izumitani 1975 Gordon Lecture.

low temperature results are to be expected as there would be a hardness below which the ceria would not be expected to break even the hydrolized layer. The more significant point is that Isumitani found a decrease in rate above a firing temperature of 900°C. He was quite puzzled by this and speculated that the particles might be more brittle, breaking down more easily. I suspect both sets of data are correct, and that the answer lies in the preparation of the material which can be prepared from a number of sources, nitrates, phosphates and oxalates, as well as ground by different processes to different grades of fineness.

My purpose in this digression into chemistry is to point out that different polishing agent processes even for the same chemical material may yield different results when compounded with pitch variations. We may not be able to isolate pitch response even from the polishing compound preparation.

Returning to the main topic, pitch interactions, we have two striking figures from Izumitani that I have several times referred to in passing. These are from Ref. 1.

These show the effect of creep compliance at 60 seconds on the polishing rate of soft SF6 glass and medium BK7 in Figures 23 and 24 respectively. The abscissa is mislabeled on the figures in the report but correctly stated as seconds in the text. Since the term compliance is used, I should explain that this is not compliance in the sense that we have used it. Here shear moduli and shear stress compliance are referred to, the compliance being the reciprocal of an effective shear modulus determined after 60 seconds of creep.

$$\tau = \mu \frac{\Delta Y}{h} = \frac{\mu \Delta L}{\Delta T h} = {}^{u}G^{u} \frac{\Delta L}{h}$$

$$\therefore \frac{1}{{}^{u}G^{u}} = \frac{\Delta T}{u}$$

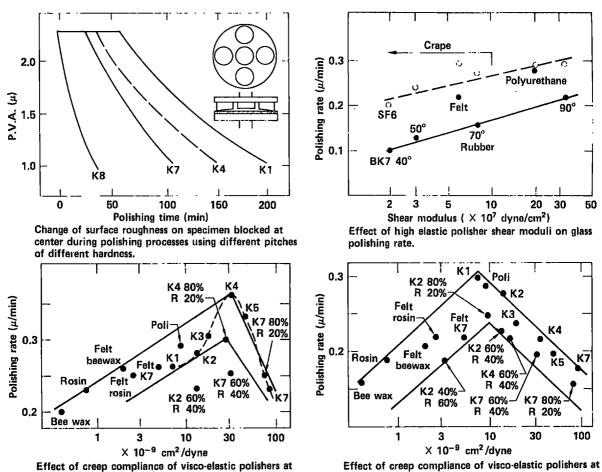
Thus these are really a type of viscosity and we have here experimental verification of the old opticians adage "soft glass, soft pitch; hard glass, hard pitch". The optima according to the report fell at 30 x 10^{-9} cm²/dyne yielding 2 x 10^{9} poise for SF6 a soft glass, and at 8 x 10^{-9} for BK7 yielding 7.5 x 10^{9} poise for BK7.

Dr. Izumitani has sent me viscosity measurements made on pitches Kl through K7 by himself and Harada and also by Wada and Hirose. These are

	Izumitani & Harada (23°C)	Wada & Hirose (25°C)
K1		1.4 x 10 ¹²
K2	1 x 10 ¹¹	5.2 x 10 ¹¹
K3	1 x 10 ¹⁰	1.6×10^{11}
K4	3 x 10 ⁹	4.8×10^{11}
K5	2 x 10 ⁹	9.9 x 10 ⁸
K7	1 x 10 ⁹	

The measurements were taken a decade apart which can easily describe the differences between the two sets. Izumitani's more recent data agrees well with the creep compliance data for K3 through K7 but not for K2, and the earlier work seems very far off for K1. Very slight additions of volatiles would bring it into line however. Even so when a line is drawn through the K3 to K7 points (which are consistent) a very distinct optimum appears on the SF6 curve. trend also seems well established on the BK7 curve. The SF6 curve clearly shows almost a factor of two rate difference between the K4 and K7 pitch values and a half width of about half a decade to a decade at the half width points. For pitch with its exponential temperature variation this is a rather narrow zone corresponding to between three and four degrees centigrade; we had generally found this to be two to four degrees centigrade for planetary lapping rather heuristically without quantifying the effect as reported in Ref. 9.

At this point, I am almost certain that for any glass there is a pitch too hard since the particles



Ref. HGW-0-3E

Figures 21, 22, 23 & 24: Effect of elasticity and viscosity on polishing rate from Izumitani, Ref. 1. Note K1 through K7 are pitches.

cannot properly socket themselves. For this effect a softer pitch is required for a softer glass, because the particles tend to dig into the soft glass more deeply tending to produce a greater shearing force requiring deeper socketing.

60 seconds on polishing rate of SF 6 glass.

The drop-off on the softer side which is most clearly pronounced and incontrovertable is in my opinion due to a greater tendency for the particles to roll in too soft a socket exposing less preferential cutting edges. However, I must point out that a greater ease of plowing is quite as good an explanation, and another possibility is sinking more easily into the pitch although there are some difficulties with this last case since other particles should take up the load and cutting.

At any rate, the important point to note is that there does appear to be an optimum. These data are incomplete on several counts. Both SF6 and BK7 are of the silicate family and the two Figures 17 and 18 taken from the same Hoya report tend to show a significantly different polishing rate behavior for the borate family of glasses. In addition, the effect of polishing compound selection on this phenomenon has not been studied. Work in both these areas is in progress.

Furthermore, the data do not tell us whether we want to work at the optimum in all cases since we have hydrolysis rates and surface smoothness to consider. However, the economic implications are obvious and this is certainly something to look for in setting up production.

60 seconds on polishing rate of BK 7 glass.

We have made another set of observations that bears on the point of particle penetration, involving edge roll-off on continuous flat laps. Since this can be on the order of microns, it is obvious that this is not an elasticity effect or a time dependent particle sinking into the pitch from calculations made earlier in this section.

We hypothesized that a large particle entering beneath the glass could score deeply for a while, plowing through the pitch either continuously or after heating. Large particles sprinkled on the lap penetrated a distance inversely to their diameters, and reducing the facet size significantly (to decrease the plowing distance) greatly reduced the roll-off problem.

A similiar phenomenon bearing on glass pitch interactions is scratching on planetary laps. It has

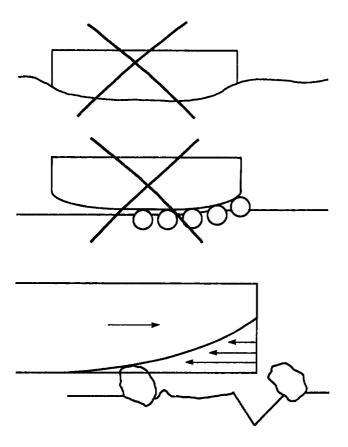


Figure 25: Edge roll-off on flat planetary laps is not due to elastic deflections or to sinking particles, but to various sized particles abraiding the edge for short distances.

been noted previously (Ref. 9) that shops employing planetary laps find polished chamfers greatly reduce the incidence of scratching. In studying scratches, we always found them associated with an edge breakout at the apparent origin of the scratch. More recently we have been studying a soft glass that is quite prone to scratching. Square samples prepared with polished edges and chamfers on opposite sides, and fine and coarse ground chamfers on opposite sides typically show no scratches originating on the polished chamfer sides, a few on the fine ground side, and noticeably more on the coarse ground edge. For a long time I believed these were due to chips breaking out of the edge, but I believe evidence is accumulating that these are due to large particles or clumps jamming into a socket in the glass and against the pitch, originating a further breakout and progressing across to scratch. This mechanism is consistent with the grinding studies of Saccocio (Ref.10) who found that the edge serves as a size discriminating selector. At first glance this seems inconsistent with the above observations on edge round off which seem to indicate the largest particles should not penetrate the interface very far. There are several possible explanations to account for this. First, the edge breakout indicates the process is initiated with considerable violence indicating that the scratching

particles could be jammed into the pitch so deeply that plowing simply cannot take place. Secondly, if heating of the particles influences plowing, there may be a size where the heat capacitance is such that the particle cannot heat up sufficiently rapidly to begin plowing. As noted earlier, the disruption energy should be proportional to the particle size squared but its heat capacitance should vary as the cube, increasing the time to reach a certain temperature by the size of the particle.

Grooves and facet edges are important, and the more edges the better for fast even cutting. Fluid turbulence in the grooves, easily visible through the glass when pitch chips are present continuously redistributes polishing particles, and facet edges present them in new orientations to the glass, more edges increasing the probability of a cut. Pitch pressed against window screen produces a superior cut. A fresh cut and rough scraped lap may have a coefficient of friction (a measure of cutting) of 0.75 to 0.8. After some hours when the pitch has become smooth, rubbery, dead, this may drop to 0.30 and even to 0.17. For a while until the surface becomes fully charged with particles, wire brushing or dragging backsaw blades crossways across the pitch may restore the lap.

SUMMARY

In this section we have described a number of pitch slurry interactions that obviously have a significant effect on polishing rate and cosmetic quality. The economic importance is obvious, but even here care must be taken not to overstate the case. Polishing rate is not the only factor in finishing. Uniformity and control of figuring are of major importance obviously and so is cosmetic quality. These may not be consistent with maximum possible polishing rate.

Work is in progress to investigate these areas but definitive results may be a long time coming. For some shops optimization along these lines now is a factor of importance.

All we can suggest for now for those wishing to pursue the subject is to polish small samples on a small ring planetary lap with the same slurry and different pitches. Weighing a 50 to 100 gm sample to a milligram or a tenth of that is comparatively simple on a good microbalance from which weight changes corresponding to thickness changes on the order of $\lambda/100$ may easily be inferred, permitting a shop to optimize its own pitch slurry combinations.

ACKNOWLEDGEMENTS

It is obvious that a report of this nature covers the work of numbers of people and organizations and is built up on the work of previous investigators most of whom have been acknowledged in the report. Special thanks are due to Dr. William Mason of LLL who performed the computations and to Dr. Tetsuro Izumitani of Hoya Glass Works for a number of extremely fruitful discussions covering topics beyond those acknowledged.

Whatever practical reality is contained herein is principally due to the observations of Dr. Gregory Sanger and the optician staff at LLL, Phil Baker, Dick Maney, Don Murrell, Eberhard Prochnow, Dick Stolcis and Wayne Whistler.

APPENDIX A PITCH TESTING

From the foregoing, it is obvious that the primary property of interest to the optician is pitch viscosity. This Appendix will address the practical measurement of this property.

The obvious first source of testing procedures is the American Society for Testing Materials (ASTM). They provide two test methods for high viscosity materials. The best is covered in ASTM publication D-3205-74, Tentative Method of Test for Viscosity of Asphalt with Cone and Plate Viscometer. This is used for the range of 10^3 to 10^{10} poise but requires calibration using standard oils available only in the range from 10^3 to 10^4 poise. This is quite a ways from the range of 10^8 to 10^{11} poise in which we are interested. I know of at least one large shop that has used this method, but they claim poor results.

ASTM publication D5-73, Standard Test for Penetration of Bituminous Materials, covers a needle type test somewhat similar to the Twyman test, but adequate correlations are not available.

Faced with this situation we developed a form of Searles cylindrical tester. This consists of a rotating cylinder placed within a fixed cylinder with pitch filling the annulus between. The inner cylinder is held in position with sealed ball bearings and torque is applied to the inner shaft by weights connected over a pulley to a drum fastened to the inner shaft. The entire apparatus is immersed in a temperature controlled water bath. The unit is not completely frictionless. A weight hanger was progressively cut down until zero rotation appeared overnight, with a final weight of approximately 50 grams, small in comparison to the test loads which ranged from one to five kilograms.

The formula for determining the viscosity with such an apparatus can be developed as follows, letting "a" equal the inner radius, "b" the outer radius, "£" the cylinder length, "T" the applied torque, and " μ " is the viscosity.

$$\tau = \mu \frac{dv}{dr} = \frac{T}{rA} = \frac{T}{2\pi r^2 \ell}$$

$$\int_a^b dv = \frac{T}{2\pi \mu \ell} \int_a^b \frac{dr}{r^2} = \frac{T}{2\pi \mu \ell a} \left[1 - \frac{a}{b} \right] = \omega(b-a)$$

$$\mu = T[1-a/b] / 2πωℓa(b-a)$$

If all dimensions are in centimeters, force in dynes (load in grams x 981) and ω the angular velocity is in radians per second, the units of viscosity will be in poise (dyne sec/cm²).

This method is probably too complex for normal shop use, but it is by far the most accurate and repeatable, and it is highly recommended for any serious pitch research or for acquiring data for advanced automated processes.

Penetrometer testing requires a clean fresh pitch surface. We had erratic results until we found a proper surface handling procedure. We pour our pitch sample (stirred just prior to pouring) onto a piece of aluminum foil, shiny side up using a tape dam. The sample thickness is usually one to two centimeters. Before the pitch hardens we place a small round of work glass on the upper surface while the pitch is still soft enough to make a good bond to the glass. Bubbles rise away from the foil which serves to protect the surface from oxidation and perhaps prevent more volatile components from diffusing out of the surface. When this cools sufficiently we place the sample in a water bath and after it has come to equilibrium we remove the foil which strips off cleanly with no need for surface preparation. We then carefully wring the glass onto the platform under the water and lower the ball onto the pitch, starting a stopwatch. Depth ponetration is measured with a dial indicator, those we use having a one inch travel and having half mil divisions with readings to tenths being easily estimated. A term easily overlooked in this procedure is the dial indi-cator load which can be appreciable. These are spring loaded to insure proper following action. For example, the indicator I am presently using has a load of 37 grams necessary to begin deflection and an increasing load of 200 grams per inch of deflection. This is easily measured by up-ending the indicator and placing known weights on the indicator arm. We use the average of the initial and final indicator load added to the separate weights.

The attention to pitch surface preparation for this test may lead some to question its validity for predicting flow characteristics on laps whose surfaces are not so carefully treated. There is some merit to this question. However, as we have seen, lap flow characteristics are determined by pitch flow well beneath the surface. In this test, however, we are testing flow at, or very near the surface and surface effects can have a very strong effect.

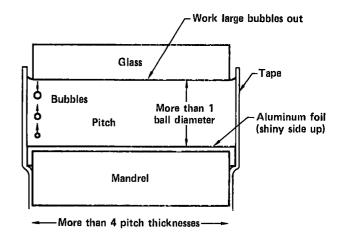


Figure 26: Preparation of pitch samples for penetrometer testing.

In his 1922 paper, Preston recommended a ball test for viscosity stating that

$$\mu = C \frac{wt}{h/hd}$$

the form of the equation we have seen several times. This test is not widely used in this country presumably because penetrations are shallower requiring more precise readings, and because it is sensitive to surface preparation. With the dial indicators now available, I believe it is a more practical test for small shops than the truncated cone test for most work.

The form of the equation has been verified by the Lee and Radok work and we have verified this for highly greased balls penetrating less than 1/4 of the ball diameter, using balls ranging from a quarter inch to an inch, for loads ranging from a few hundred grams to several kilograms and for viscosities ranging from 5 x 10^8 to 5 x 10^{10} .

We are tentatively using the following formula.

$$\mu = 2.2 \times 10^4 \left(\frac{\text{wt h}^{-1.5}}{\sqrt{D}(1-1.4h/D)} \right)^{-*}$$

where h = penetration depth cm

t = time in minutes

w = weight in grams

D = ball diameter in cm

 μ = viscosity poise

The constant is (981 x 60 x .374) containing the acceleration of gravity, minutes to seconds conversion, and a constant just twice that in the Lee and Radok formula. I do not know the source of the discrepancy. The (1-1.4h/D) factor is necessary to preserve the original form of the equation

$$a^{3}(t) = \frac{3DF}{16} \left[\frac{1}{E} + \frac{t}{\mu} + \chi(t) \right]$$

where a is the radius of the indentation. The removal of this factor is an approximation valid only for h<<D. I do not know where the formula falls apart but it appears to work well for values of h below one quarter of the diameter.

A similarly tentative equation for the Twyman penetrometer, a 14 degree cone truncated at $1/2~\mathrm{mm}$ is

$$ln\left(\frac{wt}{\mu}\right) = ln(10h) + 0.009037 ln^{3\cdot3}(10h) - 15.75$$

for 0.10 mm <h< 10 mm for h<0.1 mm omit $1n^{3\cdot3}$ term

$$ln(\frac{wt}{\mu}) = 2(n(10h) - 18.96)$$

for h<10 mm

Note! Here h = mm

The symbols remain the same except for h which is here kept in millimeters which opticians have traditionally used for this test. I have retained the usage since many shops have accumulated data in millimeters and many opticians have some feel for viscosities in terms of millimeter penetrations.

Both tests become more consistent if the penetrometers are lightly greased with silicone vacuum grease. Pitch is sticky at the most awkward times and irregular and unpredictable adhesion to the metal balls or cone appears to have an effect.

I am confident of the form of the equations but not in the constants 2.2, and 0.7 in the ball equation and the 15.75 and 18.96 in the cone equation. These stand or fall together. I had believed they were better than they were till shortly before publication when some awkward data pricked the balloon. They appear to agree with the cylinder tester to about 25%, and of the two the ball tester is considerably more consistent.

I suspect three problems with the Twyman test. Both tests are sensitive to initial reading, but it is more difficult to get a good zero on the Twyman because of the high tip loading on the initial penetration. The small area is also sensitive from the standard of inhomogeneities, and finally the steep slope is just begging for the pitch to stick. The slope of the plot of the Twyman penetration on log-log paper gradually changes from unity to two over the two orders of magnitude plotted. Thus penetration behavior is the type for a flat plate initially gradually becoming that of a cone as penetration progresses.

Dispite a preference for the ball test, I find both tests are valuable. Some of the harder materials are almost impossible to test with a ball and many of the softer materials are almost impossible to test with the cone. I recommend a penetration tester set up to use both balls of various sizes and the cone tester simply by screwing on different tips.

The photograph in Figure 28 shows an assemblage of viscosity testing equipment whose use should be obvious.

A final note should be made on accuracies to be expected. Measurements beyond 25 to 30% are quite difficult with this type of temperature sensitivity. Furthermore, our processes are not that sensitive or we could not have worked effectively in the past. The peak of the sharp dotted curve in Figure 23 showing the effect of viscosity on polishing SF6 glass is plotted with a logarithmic abscissa. On the other hand the distance from the K4 to K7 points on this curve is only a factor of three, so we cannot be careless. The implications here are that about three degrees centigrade or five degrees fahrenheit mean almost a factor of two in polishing rate and that is a lot of dollars. This may seem a fairly large temperature range in shops priding themselves on their air conditioning, but work can range from a number of degrees above the room set points in high speed processes to several degrees below in slow processes affected by evaporation and humidity.

*SEE note at top of Table 8 Appendix D!

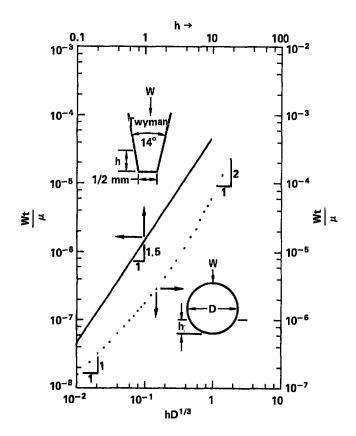


Figure 27: Comparison of ball and Twyman penetrometer test results. Note that each curve employs a different set of scales.

APPENDIX B CONSIDERATIONS ON PITCH LAP COMPLIANCE

This appendix will consider several topics more speculative than are proper in the body of a technical report.

Consider first the edge of the lap. The simplest equation shows that basically flow varies as the cube of thickness. In a controlled flow process we would have the center of the lap flowing as described. At the edges, however, we have a larger unsupported thickness and consequently much more rapid flow. There will be a tendency for the pitch to become thinner at these edges or conversely to exert considerably less pressure in a fitting operation. Now a top optician frequently designs his backing plate so that at least initially the pitch is thinner at the edge. One reason given is that this is an allowance for average pitch flow during the process, that the optician desires a uniform thickness (i.e., backing tool concentricity to the part) at the final stages of figuring. I submit that one other reason this works is that flow is rendered more uniform at the initial stages. Now a man might want the pitch edge thickness to increase or decrease toward the end of life. I submit that a clamp of the Aeroseal type might be raised or lowered on a lap to achieve this.

Next consider the case of a tool working a piece of glass from flat to a slight convexity, using an overedge stroke. For simplicity assume both tool and glass are the some diameter. As the center loaded tool passes over the edge a non uniform pressure distribution results across the interface (Figure 28) producing greater pressure averaged over the center of the tool and over the edge of the workpiece than in other areas. Since only truly spherical shapes can mate at all contact points, a spherical surface will result, the workpiece going convex. For small departures from the plane, this can be approximated as a parabola. This is actually the case for any small change in sphericity where the radius change is small, the cases the optician would work in this fashion, the cylinder described below being a thin spherical layer in this case. This parabolic sag can be defined by a hypothetical cylinder passing through the edges and peak. The volume enclosed by the sag surface is exactly half the cylinder volume. Thus the volume removed from the tool and that from the workpiece in this case by non uniform pressures must be identical. There will also be a uniform wear term on each surface due to the uniform pressure component.

Now either the tool or the workpiece can have a compliance greater than the other. We certainly do not wish the tool compliance to be less than the glass compliance or the tool would retard the curvature change we presumably wish to make. I maintain that equal compliance is the optimal case. It may be argued that since we are restricted to the glass compliance induced rates of curvature change, a more compliant tool is satisfactory. This may be true for a while, but certainly there will come a degree of tool compliance (i.e., jelly) where the tool cannot locally produce or support pressure differences sufficient to effect the change. Furthermore, to obtain a maximum degree of regularity, we usually prefer as stiff a lap or as low a compliance in the pitch pad as is consistent with the process. This occurs in this case when compliances are equal.

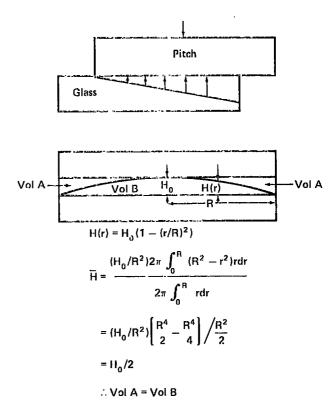


Figure 28: Result of a convexing stroke.

To make use of this relationship, we must be able to compute the glass compliance values. These are not available as tables since wear rates as mentioned at length are functions of glass, slurry, and pitch to a degree just now becoming clear. However, any shop can determine the glass wear rates and should certainly do so for its most common material combinations. method for doing so is described at the end of the main section on pitch slurry interactions. When this is done. The shop will have an equation in the form of the Preston wear equation

where H = rate of thickness removal

P = pressure

V = mean velocity

C = process constant

H/P = glass compliance

Then

$$\frac{\ddot{I}}{D} = CV$$

The problem is to determine \overline{V} , the mean velocity on the lap. This has been done to 10% accuracy by Sergeev and Golovanova (Ref. 6, pg. 100) as

$$\overline{V} = 2\pi n_L L = 7$$
 when $i = 3$

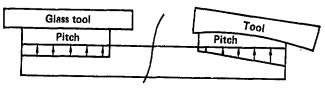
$$\overline{V} = Z \frac{2(i^2+1)}{3i^2+1}$$
 when $i > 1$
 $\overline{V} = Z \frac{2(i^2+1)}{i(3+i^2)}$ when $i < 1$
and $i = n_L/n_K$

where n_L = number of lap revolutions per unit time n_K = number of crank revolutions per unit time L = half the length of the center symmetrical stroke.

For planetary laps, i is usually l and L is the distance from the lap center to the center of the ring position.

Now in the case just considered the compliances turned out to be the same. This is so only because the lap and tool areas were identical and further because exactly equal proportions of each area were always in contact.

To demonstrate my meaning and carry our reasoning further, consider the case of a single planetary lap (Ref. 8). Here the operator is continually varying the position of a large glass tool moving it in and out to effect concave or convex changes to lap figure. This tool is usually exactly half the lap diameter. The operator certainly does not want the lap compliance to be less than the tool compliance as this would retard the "control" changes he is trying to accomplish. Furthermore, if the lap compliance is much greater than the minimum required, the outward movement of the flat tool will tend to make the lap try to go conical for a while rather than convex spherical as desired. To determine the proper compliance we shall examine an actual case in planning.



Lap diam $\approx 2 \times$ tool diam

Lap area $\approx 4 \times$ tool area

Lap sag $\approx 4 \times$ tool sag

Lap dwell $\approx 1/4 \times$ tool dwell

Lap compliance $\approx 64 \times$ tool compliance

Tool compliance =
$$\frac{\Delta H}{P\Delta T}$$
 = V X f(glass, slurry, pitch)

Lap compliance =
$$\frac{\Delta H}{P\Delta T}$$
 = f(μ , shape, H, W, D)

Figure 29: Single planetary lap case.

We have a new glass, soft and temperature sensitive in the extreme. First it is soft, so we will need a new polishing compound, the selection of which

we discussed elsewhere in this report. Having selected this we must select a pitch viscosity. This will be a soft pitch and we will head for the optimum on a curve similar to the one first published by Izumitani. This curve may be a function both of glass hydrolized layer hardness and of particle composition and size so it must be re-established for the combination. We will head for the peak but this may cut too fast for the pressure of the glass resting on the pitch leaving a rough surface by cutting right through the hydrolized layer. In this case we will back off moving toward the less viscous pitch side as this yields a better surface. Having found the viscosity we will select a temperature as close as possible to the given shop temperature for our temperature control system to minimize thermal gradient, transient, and shock effects for the thermally sensitive glass. Next we will mix two pitches of the same family to yield the desired viscosity at this temperature. We will pour that pitch on the lap to a thickness that will give us as reasonable a life as we can expect.

Now we must select the lap compliance. We will have found the glass wear rate for the constant in Prestons equation from earlier work above

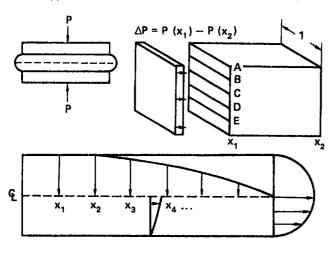
$$\mathring{H} = C \overline{PV} ; \mathring{H}/\overline{P} = C\overline{V}$$

We know the pressure due to the part resting on the pitch from the part thickness and the velocity from the machine speed. Here I am speaking of the large glass tool that forms or controls the large lap surface Ref. 8. This may or may not have to be faced with glass of identical type depending on whether the selected polishing material cuts the basic glass tool material. As stated elsewhere in this report, facet size can have an effect on edge roll-off and this glass has a roll-off problem. With this in mind we will try to select a minimum facet size (i.e., W/H is now determined).

Now we must properly select the groove depth to give us the right compliance. Due to the thermally sensitive nature of the glass we can expect thermally induced warping since even the best temperature control system cannot completely eliminate gradients and a residual linear gradient will induce spherical warping. Assume as an example, we are starting with a flat lap, and find we must make the lap convex to compensate for warping. Shifting the tool out will make it go concave and the lap go convex. We want as rapid a control change as we can get, that is we do not want the lap compliance to be too low to follow the tool figure. We will forget the central hole in the lap because the overedge portion of the tool removes about a similar portion of it from contact and say that the area of the lap is just four times the area of the tool which is exactly half the lap diameter. The sag of the lap will therefore be four times the sag of the tool for the same radius of curvature when the lap and tool are mating. Thus the volume of the cylinder enclosing the lap sag will be four times four or 16 times that for the tool after the change is effected. We have earlier pointed out that the concave and convex portions of this cylinder volume are equal. Now the tool is almost always in contact with the lap, but the different portions of the lap are in contact with the tool only a quarter of the time, so sixteen times the volume of tool material removed due to differential pressure must be removed from the pitch lap by flow in one quarter of the time exposure to identical pressure differentials.

APPENDIX C COMPUTATIONS

The computations for the section on macroscopic flow approximations are as follows for the strip:



Łooking at a little element of width one unit long, we see that the net force producing a shear across the area $\Delta x \times 1$, y distant up from the base is given by the pressure at x, P(x), minus the pressure P(x+ Δx) on the other side acting on the area (h-y) x l. This will produce a slight change in velocity going from one side of the plane at y to the other. We may write this equation thus

$$\frac{\tau}{\mu} = \left[\frac{P(x) - P(x + \Delta x)}{\Delta x} - \frac{(h - y)}{\mu} \right] = -\frac{(h - y)}{\mu} - \frac{dp}{dx} = \frac{dv}{dy}$$

At this point our equation is mathematically exact for one dimensional flow only. We have however a two dimensional flow situation, the second dimension of which we are attempting to minimize. Thus the student of hydraulics will note a discrepancy between this and the Navier-Stokes formulation with which he is familiar. More on this

$$\nabla \cdot \mathbf{v} = 0$$
; $\nabla^2 \mathbf{v}_{\mathbf{x}} = \frac{1}{\mathbf{u}} \frac{\partial \mathbf{r}}{\partial \mathbf{x}}$

shortly. Continuing with our equation we may integrate directly

$$v(y) = \int_{0}^{y} dv = \frac{1}{\mu} \frac{dp}{dx} \int_{0}^{y} (h-y)dy = -\frac{1}{\mu} \frac{dp}{dx} \left(hy - \frac{y^{2}}{2} \right)$$

The average velocity across location x may be determined by again integrating from zero to h and dividing by h

$$\overline{v}(x) = -\frac{1}{\mu h} \frac{dp}{dx} \int_{0}^{h} \left(hy - \frac{y^{2}}{2} \right) dy = -\frac{1}{\mu h} \frac{dp}{dx} \left(\frac{h^{3}}{2} - \frac{h^{3}}{6} \right)$$

$$= -\frac{h^2}{3u} \frac{dp}{dx}$$

Now we can write a second equation for $\overline{v}(x)$, the average velocity at x. All the material from the centerline to x that is being displaced by the plate moving toward the center of thickness must be moving across χ (i.e., h x χ x 1), flowing through a passage h thick. Therefore

$$\overline{v}(x) = \frac{h^2}{3u} \frac{dp}{dx} = \frac{hx}{h}$$

Now here is the little discrepancy we noted earlier. We are neglecting to describe how each little bit of material displaced at the plate surfaces flows from the plate and distributes itself appropriately through the cross section. This would complicate the solution with a lot of garbage that would be negligible a short distance away from the entry point in a very thin strip. Taking up our solution and integrating again we see

$$\frac{h^3p}{3\mu_h^6} = -\int xdx = \frac{x^2}{2} + c$$

Using the boundary condition P(L) = 0 we obtain

$$P(x) = \frac{3\mu \mathring{h}(L^2-x^2)}{2h^3}$$

We see that the pressure distribution is an inverted parabola peaking at the centerline. Of course the optician is generally not interested in the pressure distribution but in the mean pressure \overline{P} so carrying this one step further

$$\overline{P} = \frac{c}{L} \int_{0}^{L} (L^{2}-x^{2}) dx = \frac{c}{L} \left(L^{2}x - \frac{x^{3}}{3} \right) \Big|_{0}^{L} = \frac{2}{3} cL^{2} = \frac{2}{3} P_{\text{max}}$$

Therefore

$$h = \frac{h^3 \overline{P}}{\mu L^2}$$

A quick dimensional check shows

$$\frac{\text{cm}}{\text{sec}} = \text{cm}^3 \frac{\text{dyne}}{\text{cm}^2} \frac{\text{cm}^2}{\text{dyne sec cm}^2} = \frac{\text{cm}}{\text{sec}}$$

For the circular case we can simply replace x with r obtaining identical results for the first three steps

$$\frac{\tau}{\mu} = -\frac{dp}{dr} (h-y) ; v(y) = -\frac{1}{\mu} \frac{dp}{dr} \left(hy - \frac{y^2}{2}\right)$$

and

$$\overline{v}(r) = -\frac{h^2}{3u} \frac{dp}{dr}$$

The first change comes when we equate this to the amount of material flowing past ${\bf r}$.

$$\overline{v}(r) = \frac{h^2}{3u} \frac{dp}{d4} = \frac{\pi r^2 h}{2\pi r h} = \frac{rh}{2h}$$

yielding

$$dp = \frac{3\mu h^{\circ}}{2h^{3}} (-rdr)$$
 and $P(r) = \frac{3}{4} \frac{\mu h(r^{2}-r^{2})}{h^{3}}$

the same formula as before except for the factor of two. For the averaging, letting $P(r) = c(r_0^2 - r^2)$

$$\overline{P} = \frac{c2\pi \int_{0}^{r_0} (r_0^2 - r^2) r dr}{\pi r_0^2} = \frac{c2}{r_0^2} \left[\frac{r_0^4}{2} - \frac{r_0^4}{4} \right] = \frac{cr_0^2}{2} = \frac{P_{\text{max}}}{2}$$

$$\therefore \hat{h} = \frac{8}{3} \frac{h^3}{\mu} \frac{\overline{p}}{r_0^2}$$

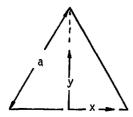
Noting the analogy between these thin slab formulae and the heat transfer equations, we can go to a heat transfer text (Ref.11) and obtain the solution for squares and rectangles.

$$T = \frac{A_0}{K} \left(\frac{(a^2 - x^2)}{2} \right)$$

$$-\frac{16a^{2}}{\pi^{2}}\sum_{n=0}^{\infty}\frac{(-1)^{n}\cos\left[(2n+1)\frac{\pi x}{2a}\right]\cosh\left[(2n+1)\frac{y}{2a}\right]}{(2n+1)^{3}\cosh\left[(2n+1)\frac{\pi b}{2a}\right]}\right)$$

$$T_{\text{max}} = \frac{A_0}{K} \left(\frac{a^2}{2} - \frac{16a^2}{\pi^2} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^3 \cosh\left[(2n+1)\frac{\pi b}{2a}\right]} \right)$$

The solutions for triangles is harder to come by, but I am indebted to Dr. William E. Mason for a most elegant solution for an equilateral triangle of side a.



$$T = \frac{A_0}{K} \left(a^2 \frac{y \left[(y - \sqrt{3}/2)^2 - 3x^2 \right]}{2\sqrt{3}} \right)$$

yielding

$$T_{\text{max}} = \frac{A_0}{K} = \frac{a^2}{36}$$
, and $T_{\text{avg}} = \frac{A_0}{K} = \frac{a^2}{80}$

I should at this point explain to those not too practiced in mathematics that these formulae are not presented to snow them, nor would I expect most opticians to wade through them. These formulae can be extremely valuable to researchers exploring the field further on the computer, since they provide limiting values against which computer programs can be checked. The closed form solution for the equilateral triangle was used by Dr. Mason to check his triangular mesh size for example, guaranteeing three digit accuracy in the results shown on the next page. These are isotherm plots of temperature for the shapes of interest from a heat transfer code. The assumptions were unit rate of heat generation, unit thermal conductivity, unit dimension in the narrow direction, and edges held either insulated or at steady state zero as should be obvious. These plots of isotherms are easily transformed into the equivalent isobars from which flow studies can proceed. The data of Table 4 were derived from these. The two cases of the equilateral triangle solution having one and two legs insulated obviously transform to the diamond and hexagon respectively.

The next page is a xerox of the actual computer graphics for selected cases showing some of the pitch flow phenomena involved. They show selected velocity vectors in cross section and a normalized pressure plot above. The pressures were backtracked from an assumption of pure Newtonian flow, uniform 10^{-6} cm/sec downward velocity on the upper surface with zero horizontal tractions, and 1 cm thick pitch of 10^{10} poise viscosity for most cases.

The first shows the 64:1 width to thickness strip case. The parabolic velocity and pressure profiles are clearly evident as is the 2/3 mean to peak pressure ratio.

The next shows the same case at 32:1 aspect ratio. Here a little burble is evident on the pressure profile at the edge necessitating reruns with a graded mesh size evident in all subsequent plots.

The next is a 16:1 radius to thickness case showing the 2:1 peak to mean pressure ratio for thin circular strips quite clearly.

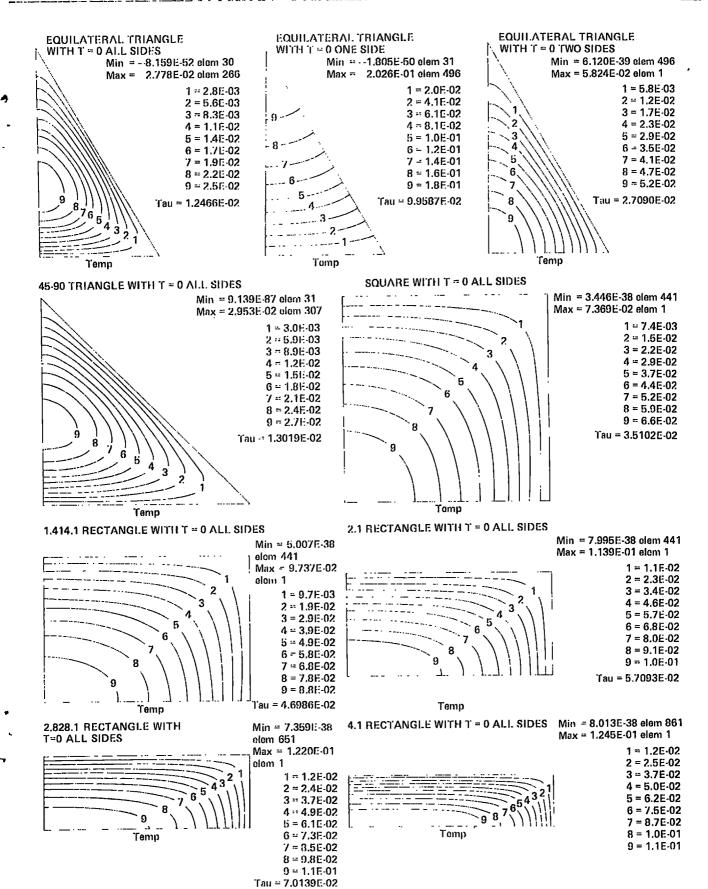
The next two show 8:1 diameter (i.e.,4:1 radius / thickness) circle cases the first for vertical cut grooves and the second with 45 degree cut grooves each to 60% of thickness.

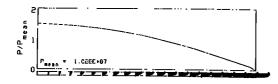
In the second column we have a 2:1 strip case with full vertical cut showing how the greater portion of the strip even with a full cut. The wide discrepancy between this and the thin strip solution should be obvious.

The next figure shows a 60% cut circular case at 4:1 aspect ratio.

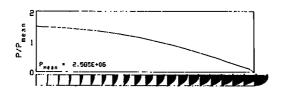
The different character of the peak to mean pressure in the two 8:1 circular cases is due to the fact that the mean pressure is calculated based on an area reaching across to the groove center. If it were to the groove edge the peak ratio would be near 1.5.



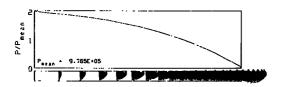




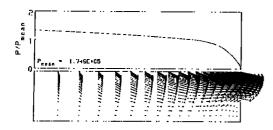
STRIP WITH 64:1 ASPECT RATIO



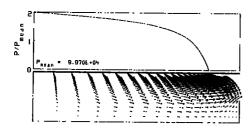
STRIP WITH 32:1 ASPECT RATIO



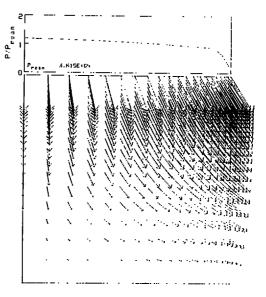
16:1 RADIUS:THICKNESS RATIO, 100% GROOVE DEPTH



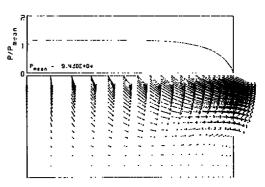
4:1 RADIUS:THICKNESS RATIO, 60% GROOVE DEPTH



4:1 RADIUS: THICKNESS, GROOVE DEPTH = 0.67



1:1 RADIUS: HHICKNESS RATIO, 100% DROOVE DEPTH



R:1 RADIUS:THICKNISS RATIC, 60% GROOVE LEPTH

Table 7
Viscosity Data

PITCH	VISCOSITY POISE	TEMP °C	POISE *	<u>K</u>
Gugolz 91	1.29 x 10 ¹¹	25.5	1.20 x 1012	0.381
	5.91 x 10 ¹⁰	28.4		
	1.03 x 1010	32.4		
	3.46 x 10 ⁹ 3.42 x 10 ⁹	35.4 35.4		
	6.11 x 108	39.4		
Gugolz 82	5.15 x 1010	24.2	2.27 x 1011	0.390
Batch A	7.92 x 109	28.6	2127 X 10	0.000
	8.25 x 10 ⁹	28.6		
	2.01 x 109	31.8		
	5.81 x 10 ⁸ 5.77 x 10 ⁸	35.0 35.0		
	9.17 x 10 ⁷	40.3		
Batch B	3.48 x 10 ⁹	30.5		
	1.96 x 10 ⁹	32.1		
	1.91 x 108	38.3		
	1.89 x 10 ⁸ 1.96 x 10 ⁸	38.3 38.3		
Gugo1z 82	4 .0	00.0		
Poorly Handled	1.65 x 10 ¹¹	23.0		
	6.96 x 10 ⁹	30.5		
	3.93 x 109	32.1		
	3.83 x 108	38.3		
Gugolz 73	7.05 x 109	22.6	1.73 x 10 ¹⁰	0.360
	7.58 x 10 ⁹ 7.41 x 10 ⁹	22.6 22.6		
	1.84 x 10 ⁹	26.3		
	4.66 x 10 ⁸	29.2		
	4.95 x 10 ⁷	35.2		
	6.30 x 10 ⁶	42.2		
Gugolz 64	5.12 x 108	23.6	1.57 x 10 ⁹	0.323
	2.79 x 10 ⁸	25,2		0,020
	1.10 x 10 ⁸	28.2		
	6.20 x 10 ⁷	29.8		
	2.31 x 10 ⁷	33.2		
Gugolz 55	1.02 x 10 ⁸	25.3	5.32 x 10 ⁸	0.317
•	5.48 x 10 ⁷	26.3	5152 X 10	0.317
	3.68 x 10 ⁷	28.1		
	1.59 x 10 ⁷	31.2		
Gugolz 73	7.85 x 10 ⁹	23.6	2.78 x 10 ¹⁰	0.359
+ Gugo1z 82 50-50	3.90 x 10 ⁹	25.4		
	3.13 x 10 ⁸	32.5		
	3.35 x 10 ⁷	38.8		
Cycad Hard	2.27 x 10 ¹¹	27.0	2. x 10 ¹²	0.311
	1.04 x 10 ¹⁰	36.9		
Cycad Soft	1.75 x 10 ⁹	24.5	5.8 x 10 ⁹	0.269
	3.02 x 10 ⁸	30.6		
	5.74 x 10 ⁷	37.2		

*For T₀ = 20°C

Table 8

Extensive data was taken on the following pitches with the ball tester. During data reduction however a number of striking anomalies occurred calling into question the calibration of the ball tester and the correlation used. This had been correlated with the Gugolz 82 and 73 pitches used in the cylinder tester using numerous samples poured from the same batches. In testing these other pitches we find the interchangeability of ball sizes and weights remains unaffected as does the consistency of readings for different penetrations of the same pitch but on different pitches the viscosity readings change with depth of penetration calling into question both the correlation and the constants. Faced with this situation, I have elected to publish room temperature readings only taken at the early stages of penetration so that shops who do no testing can have an estimate of the viscosities of pitches they are using. The pitches are all excellent products and I wish to emphasize that the deficiencies are in the tester not the pitches. Work will continue, testing each of these pitches in the cylinder tester to find the source of the problem and both better data and correlations will be published later in the Optical Shop Notebook if data warrants.

Data

<u>PITCH</u>	VISCOSITY POISE	TEMPERATURE °C
Cycad *		
(Hard.)	9 x 10 ¹⁰	22.4
Cycad	_	
(Med.)	2.4 x 10 ⁹	22.2
Cycad	_	
(Soft)	1.6 x 10 ⁹	22.3
Universal 450	4 x 10 ⁹	22.2
600	2.2×10^{12}	22.2
750	1.9 x 10 ⁹	22.5
835	2 x 10 ⁸	22.2
850	7.7 x 10 ⁷	22.2
1366	1.6 x 10 ¹⁰	22.2
Zobel		
Prec A Hard.	1011	22.1
Prec C ** Med	1.5 x 10 ⁹	22.3
Prec D ** Soft	1.5 x 10 ⁹	22.0
Opt Hard.	1.2×10^{12}	22.6
Opt Soft	7 x 10 ⁹	22.1

*The Hard Cycad pitch tested here is nearly an order of magnitude lower in viscosity than that measured in the cylinder tester. That sample was from the bottom of an old can and significant volatile fraction evaporation may have occurred.

**The Zobel C and Zobel D specimens appeared the same not only in viscosity but in temperature dependence. We suggest a shipping goof unusual for the company

which is quite reliable.

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